1. (40 Points) Consider the distribution on \( M = \mathbb{R}^3 - \{(0,0,z) : z \in \mathbb{R}\} \) generated by the following 2 vector fields:
\[
X = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \quad Z = \frac{\partial}{\partial z}
\]
Find the integral leaves of this distribution.

2. (40 Points) Prove or disprove the following statement: a Lie group \( G \) is a smooth orientable manifold.

3. (40 Points) Show that a compact connected abelian Lie group \( G \) is isomorphic to a torus \( \mathbb{T}^d \).

4. (40 Points) In \( \mathbb{R}^3 \), let \( X = x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} + z^2 \frac{\partial}{\partial z} \) and \( \omega = x^2 z dx \wedge dy + yz dx \wedge dz \). Compute the following:
   (a) \( i_X \omega \).
   (b) \( d\omega \).
   (c) \( \mathcal{L}_X \omega \).
   (d) \( \Phi^* d\omega \), where \( \Phi : \mathbb{R}^4 \to \mathbb{R}^3 \) is the map \( \Phi(u,v,s,t) = (uv,vs,t) \).

5. (40 Points) Let \( M \) be an even dimensional manifold \( \dim M = 2n \). A differential form \( \omega \in \Omega^2(M) \) is said to be non-degenerate if
\[
\wedge^n \omega := \omega \wedge \cdots \wedge \omega \in \Omega^{2n}(M)
\]
is a volume form. Show that on a compact manifold \( M \) (without boundary) a non-degenerate 2-form \( \omega \) cannot be exact, i.e., there is no \( \eta \in \Omega^1(M) \) such that \( \omega = d\eta \).