Department of Mathematics
University of Illinois at Urbana-Champaign

MATH 417 – SPRING 2017 – SECTION B1
MOCK MIDTERM 2

MARCH 10, 2017

Midterm Duration: 50 m

NAME:____

1. (25 points) Let $(A, +, \cdot)$ be a ring with identity 1_A and let $B \subset A$ be a subring with identity $1_B \neq 0$. Show that if A has no zero divisors, then one must have $1_A = 1_B$.

2. (25 points) For each of the following rings A determine if the given subset $I \subset A$ is an ideal or not. Justify your answer.
(a) $A = \mathbb{Z}$ and $I = 7\,\mathbb{Z}$;
(b) $A = M_n(\mathbb{R})$ and $I = \{A \in M_n(\mathbb{R}) : \det(A) \neq 0\}$;
(c) $A = \mathbb{R}[x]$ (the ring of polynomials with real coefficients) and $I = \{p(x) \in \mathbb{R}[x] : p(x) \text{ has constant term zero}\}$;

3. (25 points) Let D_3 be the symmetry group of an equilateral triangle. Show that the subgroup $H \subset D_3$ consisting of those symmetries which are rotations is a normal subgroup.

4. (25 points) Let $(A, +, \cdot)$ be an ordered ring. Show that A has no zero divisors.

5. (Extra Credit: 10 points) Let $\mathbb{H}$ denote the quaternions and identify $\mathbb{R}^3 \subset \mathbb{H}$ with the set of purely imaginary quaternions:

$$\mathbb{R}^3 = \{q = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} : x, y, z \in \mathbb{R}\}.$$

Show that for any fixed quaternion $q_0 \in \mathbb{H}$ with $||q_0|| = 1$, the map

$$\phi_{q_0} : \mathbb{R}^3 \to \mathbb{R}^3, \quad q \mapsto q_0 q q_0^{-1},$$

is an isometry. Find the isometry $\phi_{q_0} : \mathbb{R}^3 \to \mathbb{R}^3$ when $q_0 = \mathbf{i}$.