Department of Mathematics
University of Illinois at Urbana-Champaign

MATH 417 – SPRING 2017 – SECTION B1
MOCK MIDTERM

FEBRUARY 10, 2017

Midterm Duration: 50 m

NAME:____

1. (25 points) Let $(G, *)$ be a group such that for any $g, h \in G$ one has:

$$(g * h)^2 = g^2 * h^2.$$

Show that $(G, *)$ is abelian.

2. (25 points) Consider the permutation:

$$\sigma = (1, 7, 4)(2, 4, 6)(1, 4, 5)$$

i) Find a decomposition of σ into a product of disjoint cycles.
ii) Find a decomposition of σ into a product of transpositions.
iii) How unique are each of these decompositions?

3. (25 points) Give an example of a ring $(A, +, \cdot)$ satisfying each of the following properties (justify your answer):
(a) The cancelation law fails in A;
(b) A has non-invertible elements, but the cancelation law holds;
(c) The ring A has 16 elements.

4. (25 points) Let $(A, +, \cdot)$ be a ring with identity. Show that if $a \in A$ is an invertible element then a is not a zero divisor. Is the converse true? Justify your answer.

5. (Extra Credit: 10 points) Give an example of a non-abelian, finite, ring.