# 1. Let \((V, \| \cdot \|)\) be a Banach space. Show that there exists a compact topological space \(X\) and an isometric inclusion
\[
\iota : (V, \| \cdot \|) \rightarrow (C(X), \| \cdot \|_\infty).
\]
This shows that every Banach space can be identified with a norm closed subspace of some \(C(X)\) space.

# 2. Find all extreme points of the closed unit ball of
\[C([0,1]) = \{ f : [0,1] \rightarrow \mathbb{R}, \text{ continuous functions}\}.\]

# 3. Show that
1. there exists no extreme point in the closed unit ball of \(L^1([0,1])\),
2. \(L^1([0,1])\) is not a dual Banach space.

# 4. Let \((X, \tau)\) be a compact topological space. It is known from the Riesz representation theorem that we have the isometric isomorphism \(\mathfrak{M}(X) = C(X)^*\), where \(\mathfrak{M}(X)\) denotes the sapce of all real valued Radon measures on \(X\). Therefore, we obtain the weak* topology on \(\mathfrak{M}(X)\) under the representation
\[
\mu(f) = \int_X f(x) d\mu(x).
\]
1. Show that the set \(P^1(X)\) of all positive Radon measures of norm 1 (i.e. all Radon probability measures) on \(X\) is a weak* closed subset of \(\mathfrak{M}(X)\).
2. Show that the extreme points of \(P^1(X)\) are exactly Dirac measures \(\delta_x\) with \(x \in X\).
3. The map \(D : x \in X \rightarrow D_x \in Ext(P^1(X))\) is a homeomorphism from topological space \((X, \tau)\) onto the topological space \((Ext(P^1(X)), \tau_{w^*})\).