

Math347, Spring 2018, Homework #8
Due to Wednesday, March 28, 2018

HW 8.1. Let $\{x_n\}$ be a sequence of real numbers such that $\frac{-1}{n} \leq x_n \leq \frac{1}{n}$ for all $n \in \mathbb{N}$. Use Squeeze theorem to determine $\lim_{n \rightarrow \infty} e^{x_n}$.

HW 8.2. Let $\{x_n\}$ be a sequence of real numbers with $x_1 = 2$ and $x_{n+1} = 2 - \frac{1}{x_n}$ for all $n \in \mathbb{N}$. Show that $\{x_n\}$ is bounded and monotone decreasing. Find the limit if it exists.

HW 8.3. Let $\{a_n\}$ and $\{b_n\}$ be two sequences of real numbers given as follows

$$a_1 \leq a_2 \leq \cdots \leq a_n \cdots b_n \leq \cdots b_2 \leq b_1,$$

Show that both $\lim a_n$ and $\lim b_n$ exist, and $\lim a_n \leq \lim b_n$.

HW 8.4. Let $s_n = \frac{1}{1^2} + \frac{1}{2^2} \cdots + \frac{1}{n^2}$ for each $n \in \mathbb{N}$. Show that the sequence $\{s_n\}$ is increasing and bounded above. Is $\{s_n\}$ a convergent sequence? Explain.

HW 8.5. Show that the sequence $(1 - (-1)^n + \frac{1}{n})$ is divergent.

HW 8.6. Show that the sequence $x_n = \sin(\frac{n\pi}{4})$ is divergent.