Math347, Spring 2018, Homework #6
Due to Wednesday, March 7, 2018

HW 6.1. Let $S = \{2 + \frac{1}{n^2} : n \in \mathbb{N}\}$ be a bounded sequence of real numbers. Use definition to verify that $\inf(S) = 2$ and $\sup(S) = 3$.

HW 6.2. If $y > 0$, show that there exists $n \in \mathbb{N}$ such that $\frac{1}{2^n} < y$.

HW 6.3. Let $u > 0$ be a positive real number. Show that for any real numbers $x < y$, there exists a rational number r such that

$$x < ru < y.$$

HW 6.4. Let $S \subseteq \mathbb{R}$ be a bounded subset of $\mathbb{R}$, and let $I_S = [\inf(S), \sup(S)]$.

1) Show that $S \subseteq I_S$.

2) If J is any closed bounded interval containing S, show that J contains I_S.

HW 6.5. Prove that $\cap_{n=1}^{\infty}[0, \frac{1}{n}] = \{0\}$.

HW 6.6. Let $S = \{x : \text{irrational in } [0,1]\}$ be the set of irrational numbers in $[0,1]$. Use definition to verify that $\sup(S) = 1$.