

Math347, Spring 2018, Homework #5
Due to Wednesday, February 28, 2018

HW 5.1. Determine whether S is bounded and determine $\sup(S)$ and $\inf(S)$, if they exist.

a) $S = \{x : x^2 < 5x\}$

b) $S = \{x : 2x^2 < x^3 + x\}$.

HW 5.2. If a set $S \subseteq \mathbb{R}$ contains one of its upper bound, show this upper bound is the supremum of S .

HW 5.3. Let $S \subseteq \mathbb{R}$ be nonempty. Show that if $u = \sup(S)$, then for every number $n \in \mathbb{N}$ the number $u - \frac{1}{n}$ is not an upper bound of S , but the number $u + \frac{1}{n}$ is an upper bound of S .

HW 5.4. Let S be a bounded set in \mathbb{R} and let S_0 be a nonempty subset of S . Show that

$$\inf(S) \leq \inf(S_0) \leq \sup(S_0) \leq \sup(S).$$

HW 5.5 If $S = \{\frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N}\}$, find $\inf(S)$ and $\sup(S)$.

HW 5.6. Let (a_n) and (b_n) be two bounded sequences. Show that

$$\sup(a_n + b_n) \leq \sup(a_n) + \sup(b_n).$$