

Math347, Spring 2018, Homework #12
Due Wednesday, May 2, 2018

7.34 Find all integers that are congruent to 1 mod 7, 3 mod 8, and 5 mod 9. Which solution has the smallest absolute value ?

7.41 Let f and g be defined by $f(x) \equiv (x + a) \pmod{n}$ and $g(x) \equiv ax \pmod{n}$.

- (1) Give a complete description of the functional digraph of f for the case $(n, a) = (8, 3)$ and the case $(n, a) = (11, 7)$.
- (2) Draw the functional digraph of g for the case $(n, a) = (19, 4)$. and the case $(n, a) = (19, 3)$.

Additional Problem 1. Suppose $d = \gcd(a, n)$ and $d \mid b$. If \bar{x} is a solution for the congruence equation $\overline{ax} = \bar{b}$, show that $\bar{x}, \overline{x + \frac{n}{d}}, \dots, \overline{x + \frac{(d-1)n}{d}}$ are d -different solutions of $\overline{a \cdot x} = \bar{b}$.

Additional Problem 2. Let f be the map on \mathbb{Z}_8 defined by $f(\bar{x}) = \overline{6x}$.

- (1) Find the image $f(\mathbb{Z}_8)$ of f .
- (2) Find the inverse image $I_f(\{\bar{1}, \bar{2}, \bar{3}\})$, and the inverse image $I_f(\{\bar{0}, \bar{4}, \bar{5}\})$.