31.5 #15: \[ y' + 2xy = x \quad y(0) = 2 \]
\[ e^{x^2} y' = e^{x^2} x \]
\[ e^{x^2} y = \int e^{x^2} dx = \frac{1}{2} e^{x^2} + C \]

**General sol:**
\[ y = \frac{1}{2} + C e^{-x^2} \]
\[ -2 = y(0) = \frac{1}{2} + C \implies C = -\frac{5}{2} \]

**Particular sol:**
\[ y = \frac{1}{2} - \frac{5}{2} e^{-x^2} \]

31.6 #12: \[ xy' = y^2 + x\sqrt{4x^2 + y^2} \]
Homogeneous equation

Let \( x > 0, y > 0 \) and let \( v = \frac{y}{x} \) \( \iff \) \( y = vx \)
\[ y' = \frac{y}{x} + \sqrt{4\left(\frac{y}{x}\right)^2 + 1} = v + \frac{1}{v} \sqrt{4v^2} \]
\[ y' = x \cdot v' + v \]

Hence we get \( x \cdot v' + y = y + \frac{1}{v} \sqrt{4v^2} \)
\[ x \cdot \frac{dv}{dx} = \frac{\sqrt{4v^2}}{v} \]
\[ \int \frac{v}{\sqrt{4v^2}} \, dv = \int \frac{1}{x} \, dx + C \]
\[ (4v^2)^{\frac{1}{2}} = \ln|x| + C \quad \text{(since } x > 0) \]
\[ 4 + \frac{y^2}{x^2} = \left(\ln x + C\right)^2 \]
\[ 4x^2 + y^2 = x^2 \left(\ln x + C\right)^2 \]
31.6 #23

\[
\frac{\partial^2}{\partial x^2} x y' + 6 y = 3 x y' \quad \text{Bernoulli Equation}
\]

Let \( y' = y^4 \Rightarrow y = y^{-3} \)

\[
\frac{dy}{dx} = (3) y^{-4} \frac{dy}{dx}
\]

Then \( x (-3) y^{-4} \frac{dy}{dx} + 6 y^{-3} = 3 x (y^{-4})^{\frac{4}{3}} \)

\[
\frac{dy}{dx} - \frac{2y}{x} = -1 \quad \text{Linear Equation}
\]

Multiply \( e^{\int -\frac{2}{x} \, dx} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} \)

We get \( (x^{-2} y)' = x^{-2} \frac{dy}{dx} - 2 x^{-3} y = -x^{-2} \)

Hence \( x^{-2} y = \int -x^{-2} \, dx + c = \frac{1}{x} + c \)

\[ y^{-\frac{1}{3}} = y = x + c x^2 \]

General sol: \( y = \frac{1}{(x + c x^2)^{\frac{1}{3}}} \)

31.6 #35

\( (x^3 + \frac{y}{x}) \, dx + (y^2 + \ln x) \, dy = 0 \) \( (y > 0) \)

We have \( M(x, y) = x^3 + \frac{y}{x} \), \( N(x, y) = y^2 + \ln x \).

It is easy to see that \( \frac{\partial M}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x} \).

So the equation is exact.

\( F(x, y) = \int M(x, y) \, dx = \int (x^3 + \frac{y}{x}) \, dx = \frac{1}{4} x^4 + y \ln x + g(y) \)

Now \( y^2 + \ln x = N(x, y) = \frac{\partial F}{\partial y} = f(y) + g'(y) \)

We get \( f(y) = \frac{1}{3} y^3 + C \).

So we get \( F(x, y) = \frac{1}{4} x^4 + y \ln x + \frac{1}{3} y^3 = C \)

Implicit sol: \( 3x^4 + 4y^3 + 12y \ln x = C \)
This is a population growth model. Birth rate \( \beta = 0 \), death rate \( \sigma = \frac{1}{p} \).

\[
\frac{dp}{dt} = (\beta - \sigma)p = -\frac{1}{p} \Rightarrow p = -\sqrt{p}
\]

If \( p > 0 \), \( \int \frac{1}{\sqrt{p}} \, dp = \int -\sqrt{p} \, dt = -\sqrt{p} + C \)

So we get \( \frac{2\sqrt{p}}{p} = -kt + C \)

\( p(0) = 900 \Rightarrow 2\sqrt{900} = C \Rightarrow C = 60 \)
\( p(6) = 441 \Rightarrow 2\sqrt{441} = -6k + 60 \), \( \frac{6k}{3} = 3 \)

So \( 2\sqrt{p} = -3t + 60 \)

If \( p = 0 \), then \( -3t + 60 = 0 \) \( \Rightarrow t = 20 \)
So fish all die after 20 weeks.

32.3 #9: We have \( \frac{dv}{dt} = 1000 - 100v \)

\[
\int \frac{1000}{1000 - 100v} \, dv = \int dt + C = t + C.
\]

\[
-10 \ln |1000 - 100v| = t + C.
\]

\( 10 \ln 100 + 10 \ln |100 - v| = -t - C \).

\( (100 - v)^10 = e^C \cdot e^{-t} \) \( (C = -c - 10 \ln 100) \)

\( v - 100 = c \cdot e^{-\frac{t}{10}} \)
\( v + 10 = 100 + c \cdot e^{-\frac{t}{10}} \) \( (C = 0) \)

\( 0 = v(0) = 50 + C \) implies \( C = -50 \).

\( v(t) = 500 (1 - e^{-\frac{t}{10}}) \leq 50 \)

\( v(t) \rightarrow 50 \) as \( t \rightarrow \infty \).

So the maximum velocity it can attain is \( 50 \text{ ft/s} \).