Orbit Stabilizer Theorem.

Let G be a group and X a set. Suppose we have a group action $G \times X \to X$. The **Orbit-Stabilizer Theorem** states that for any $x \in X$,

$$|\text{Orb}(x)| = \frac{|G|}{|\text{Stab}(x)|}.$$

We have studied the symmetry groups of two dimensional object. Now we will try and learn about the symmetry groups of three dimensional objects.

Suppose P is a three-dimensional polyhedron. Let G be the symmetry group of P. Then G acts on the vertices of P by moving them according to the symmetry. We can try and determine the size of G using the Orbit-Stabilizer Theorem.

Your group has been provided with paper models of each of the following polyhedra: tetrahedron, cube, octahedron, icosahedron, dodecahedron. Use these to answer the following questions.

1. Let T be a tetrahedron, a solid with four triangle faces.
   (a) Fix a vertex of T. Determine the size of the orbit of this vertex.
   (b) Fixing the same vertex, determine the number of symmetries of T that fix this vertex. (This is the size of the stabilizer of the vertex.)
   (c) Determine the size of the symmetry group of T.
2. Now let C be a cube. Determine the size of the symmetry group of C.
3. Determine the symmetry group of a regular octahedron, a solid with eight triangular faces.
4. Determine the symmetry group of a regular icosahedron, a solid with twenty triangular faces.
5. Determine the symmetry group of a regular dodecahedron, a solid with twelve pentagonal faces.
6. Determine the symmetry group of a soccer ball, which has 12 pentagonal and 20 hexagonal faces.