HW3 is on my homepage now. I have also setup HW3 in the course Moodle page. Please submit your homework via the Moodle page. Make sure you follow the instructions there. Make sure that your HW is uploaded successfully.

Solutions to HW2 is on my homepage.
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Important! Review the student instructions at https://cbtf.engr.illinois.edu/cbtf-online/students. This includes information about making reservations, setting up your phone for Zoom (required), getting help (there are daily CBTF student office hours on Zoom), and also setting your DRES accommodations so you get their extended time during the exams.
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In all the examples of last lecture, the random variables can take at most countably many values. We describe the random variables by listing all their possible values and the probability they take these values. This does not always work.

**Example 5**

A number is chosen randomly from $(0, 1)$. Let $X$ be the value of the number.

$X$ is a random variable. Its possible values are in $(0, 1)$. The probability that it takes any value in $(0, 1)$ is 0. For any sub-interval $A$ of $(0, 1)$,

$$P(X \in A) = |A|,$$

when $A$ denotes the length of the interval $A$. 
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For a random variable $X$, the function

$$F(x) = P(X \leq x), \quad x \in \mathbb{R},$$

is called the (cumulative) distribution function of $X$.

It is a non-decreasing, right-continuous function with

$$\lim_{x \to \infty} F(x) = 1, \quad \lim_{x \to -\infty} F(x) = 0.$$

The distribution function $F$ of a random variable $X$ contains all the statistical info about $X$. If we know $F$, then we can find the probability of any event defined in terms of $X$. For instance, for any $a < b$,

$$P(X \in (a, b]) = F(b) - F(a).$$
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A random variable that can take at most countably many values is called a discrete random variable. For a discrete random variable $X$, the function

$$p(x) = P(X = x), \quad x \in \mathbb{R},$$

is called the probability mass function of $X$.

If $X$ can only take the values $x_1, x_2, \ldots$, then

$$p(x_i) > 0, \quad i = 1, 2, \ldots,$$

and

$$p(x) = 0 \quad \text{for all other values of } x$$

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$$F(x) = \sum_{x_i \leq x} p(x_i).$$

If $X$ is a discrete random variable whose possible values are $x_1, x_2, \ldots$, then in general, its distribution function is a step function. One can read off the probability mass function from the distribution function.

For instance, if $X$ is a discrete random variable with probability mass function

$$p(1) = \frac{1}{4}, \quad p(2) = \frac{1}{2}, \quad p(3) = \frac{1}{8}, \quad p(4) = \frac{1}{8},$$

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Outline

1. General Info
2. 4.1 Random Variables
3. 4.2 Discrete Random Variables
4. 4.10 Properties of Distribution Functions
Suppose that $F$ is the distribution function of a random variable $X$, then

1. $F$ is non-decreasing, i.e, $a \leq b$ implies $F(a) \leq F(b)$;
2. $\lim_{x \to \infty} F(x) = 1$, $\lim_{x \to -\infty} F(x) = 0$;
3. $F$ is right-continuous, i.e, for any $b \in \mathbb{R}$, $\lim_{x \downarrow b} F(x) = F(b)$.

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Any function on $\mathbb{R}$ satisfying the three properties above is the distribution function of some random variable.
Once we know the distribution function $F$ of a random variable $X$, we can find the probability of any event defined in terms of $X$. Here are some examples:

$$P(a < X \leq b) = F(b) - F(a), \quad P(a \leq X \leq b) = F(b) - F(a^-),$$
$$P(a \leq X < b) = F(b^-) - F(a^-), \quad P(a < X < b) = F(b^-) - F(a),$$
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Example

The distribution function of a random variable $X$ is given by

$$F(x) = \begin{cases} 
0, & x < 0, \\
x/3, & 0 \leq x < 1, \\
x/2, & 1 \leq x < 2, \\
1, & x \geq 2.
\end{cases}$$

Find (a) $P\left(\frac{1}{2} \leq X \leq \frac{3}{2}\right)$; (b) $P\left(\frac{1}{2} \leq X \leq 1\right)$; (c) $P\left(\frac{1}{2} \leq X < 1\right)$; (d) $P\left(1 \leq X \leq \frac{3}{2}\right)$; (e) $P\left(1 < X < 2\right)$. 
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