Fifth Homework Set — Solutions
Chapter 4

Problem 4.40 Let X be the number of correct answers. Then

$$P\{X \geq 4\} = P\{X = 4\} + P\{X = 5\} = \binom{5}{4}\frac{1}{3^4} \cdot \frac{2}{3} + \frac{1}{3^5} = \frac{11}{243}.$$

Problem 4.42 See part (a) of Example 6f in the book.

Problem 4.45 Let A be the event that the student has an 'on' day, and let E_3, E_5 be the event that a majority of a panel of three (resp. five) examiners passes him. Then

$$\begin{aligned}
P(A) &= \frac{1}{3}, P(A^c) = \frac{2}{3} \\
P(E_3|A) &= \binom{3}{2}0.8^2 \cdot 0.2 + 0.8^3 = 0.896 \\
P(E_3|A^c) &= \binom{3}{2}0.4^2 \cdot 0.6 + 0.4^3 = 0.352 \\
P(E_5|A) &= \binom{5}{3}0.8^3 \cdot 0.2^2 + \binom{5}{4}0.8^4 \cdot 0.2 + 0.8^5 = 0.9421 \\
P(E_5|A^c) &= \binom{5}{3}0.4^3 \cdot 0.6^2 + \binom{5}{4}0.4^4 \cdot 0.6 + 0.4^5 = 0.3174 \\
P(E_3) &= P(E_3|A)\, P(A) + P(E_3|A^c)\, P(A^c) = 0.5333 \\
P(E_5) &= P(E_5|A)\, P(A) + P(E_5|A^c)\, P(A^c) = 0.5256
\end{aligned}$$

The student would be marginally better off with three examiners.

Problem 4.48 Let p be the probability that a single package contains more than one defective diskette. Then $p = 1 - 0.99^{10} - 10 \cdot 0.99^9 \cdot 0.01 = 0.0043$, and the probability of returning exactly one of three packages is $\binom{3}{1}p(1-p)^2 = 0.0127$.

Problem 4.50 Let F be the event that six of the first ten coin tosses come up heads.

(a) $P(H,T,T|E) = \frac{P(H,T,T \text{ and } E)}{P(E)} = \frac{p(1-p)^2\binom{7}{5}p^5(1-p)^2}{\binom{10}{6}p^6(1-p)^4} = \frac{\binom{7}{5}}{\binom{10}{6}} = \frac{1}{10}$

(b) $P(T,H,T|E) = P(H,T,T|E) = \frac{1}{10}$

Problem 4.55

$$\begin{aligned} P(\text{no errors}) = & \quad P(\text{no errors}|\text{first typist})\, P(\text{first typist}) \\ & + P(\text{no errors}|\text{second typist})\, P(\text{second typist}) \\ = & \quad \frac{1}{2}\left(\frac{3^0}{0!}e^{-3} + \frac{4.2^0}{0!}e^{-4.2}\right) \\ = & \quad \frac{1}{2}\left(e^{-3} + e^{-4.2}\right). \end{aligned}$$

Problem 4.57 X is Poisson with parameter $\lambda = 3$.

(a) $P\{X \geq 3\} = 1 - P\{0\} - P\{1\} - P\{2\} = 1 - e^{-3}\left(1 + 3 + \frac{9}{2}\right) = 0.5768$.

(b) $P\{X \geq 3|X \geq 1\} = \frac{P\{X \geq 3\}}{P\{X \geq 1\}} = \frac{P(X \geq 3)}{1-e^{-3}} = 0.6070$.

Problem 4.59 Let X be the number of times you win a prize. Then X is binomial with $n = 50$ and $p = \frac{1}{100}$, i.e., we can use the Poisson approximation with $\lambda = 50 \cdot \frac{1}{100} = \frac{1}{2}$.

(a) $P\{X \geq 1\} = 1 - P\{X = 0\} = 1 - e^{-\frac{1}{2}} = 0.3935$

(b) $P\{X = 1\} = \frac{1}{2}e^{-\frac{1}{2}} = 0.3033$

(c) $P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-\frac{1}{2}}\left(1 + \frac{1}{2}\right) = 0.0902$

Problem 4.61 Let X be Poisson with parameter $\lambda = 1000 \cdot 0.0014 = 1.4$. Then $P\{X \geq 2\} = 1 - P\{X = 0\} - P\{X = 1\} = 1 - e^{-1.4}(1 + 1.4) = 0.4082$.

Problem 4.63 Let X be a Poisson random variable with parameter $\lambda = \frac{5}{2}$. Then X gives a reasonable description of the number of people entering the casino between 12 and 12:05.

(a) $P\{X = 0\} = e^{-\frac{5}{2}} = 0.0821$

(b) $P\{X \geq 4\} = 1 - e^{-\frac{5}{2}}\left(1 + \frac{5}{2} + \frac{25}{8} + \frac{125}{48}\right) = 0.2424$

Problem 4.72 Let A be the stronger team. $P(A \text{ wins in } i \text{ games}) = \binom{i-1}{i-4}0.6^i 0.4^{i-4}$, for $i = 4, \ldots, 7$. Hence

$$P(A \text{ wins best-of-seven series}) = \sum_{i=4}^{7}\binom{i-1}{i-4}0.6^4 0.4^{i-4} = 0.7102.$$

Similarly,

$$P\left(A \text{ wins best-of-three series}\right) = \sum_{i=2}^{3} \binom{i-1}{i-2} 0.6^4 0.4^{i-2} = 0.6480.$$

Problem 4.73 Let X be the number of games played in a match. Then $P\{X = i\} = 2\binom{i-1}{i-4}\left(\frac{1}{2}\right)^i$ for $i = 4, \ldots, 7$. Hence, $E[X] = 2\sum_{i=4}^{7} i\binom{i-1}{i-4}\left(\frac{1}{2}\right)^i = 5.8125$.

Problem 4.77 Let E be the event that right-hand box is emptied while the left-hand box still contains k matches. Then, using a negative binomial random variable with $p = \frac{1}{2}$, $r = N$, and $n = 2N - k$, we see that $P(E) = \binom{2N-k-1}{N-1}\left(\frac{1}{2}\right)^{2N-k}$. Now the desired probability is $2P(E)$.

Problem 4.78 Let E be the event that a single drawing results in two white and two black balls. Then $P(E) = \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = \frac{18}{35}$.

Let X be the number of selections until E occurs. Then

$$P\{X = n\} = \frac{17^{n-1} \cdot 18}{35^n}.$$

Problem 4.79 (a) $P\{X = 0\} = \frac{\binom{94}{10}}{\binom{100}{10}} = 0.5223$

(b)

$$\begin{aligned} P\{X > 2\} &= 1 - P\{X = 0\} - P\{X = 1\} - P\{X = 2\} \\ &= \frac{\binom{100}{10} - \binom{94}{10} - \binom{6}{1}\binom{94}{9} - \binom{6}{2}\binom{94}{8}}{\binom{100}{10}} = 0.0126 \end{aligned}$$