1. Two fair dice are thrown. Let $E$ be the event that the sum of the dice is odd; let $F$ be the event that at least one of the dice comes up 1; let $G$ be the event that the sum of the dice is 5. Find the probabilities of following events: $E \cap F$, $E \cup F$, $E \cap F^c$ and $E \cap F \cap G$.

**Solution**
\[
P(E \cap F) = \frac{1}{6}, \quad P(E \cup F) = \frac{23}{36}, \quad P(E \cap F^c) = \frac{1}{3}, \quad P(E \cap F \cap G) = \frac{1}{18}.
\]

2. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigarettes and cigars. (a) What percentage of males smoke neither cigarettes nor cigars? (b) What percentage smoke cigarettes but not cigars?

**Solution**
Let $E$ be the event that an American male smokes cigarettes, $F$ the event that an American male smokes cigars. Then we are given $P(E) = .28$, $P(F) = .007$, $P(E \cap F) = .005$. Thus
\[
P((E \cup F)^c) = 1 - P(E \cup F) = 1 - (P(E) + P(F) - P(E \cap F)) = .7
\]
\[
P(E \cap F^c) = P(E) - P(E \cap F) = .23.
\]

3. Three fair coins are flipped. Let $A$ be the event that the first flip equals the second flip; let $B$ be the event that the first flip is different from the third flip; and let $C$ be the event that the second flip is different from the third flip. Find $P(A)$, $P(B)$, $P(C)$, $P(A \cap B)$, $P(A \cap C)$, $P(B \cap C)$, $P(A \cap B \cap C)$.

**Solution**
$P(A) = P(B) = P(C) = \frac{1}{2}$, $P(A \cap B) = P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = \frac{1}{4}$.

4. Two fair dice are rolled. Let $X$ be the product of the two dice. Find the probability mass function of $X$.

**Solution**
\[
p(x) = \begin{cases} 
\frac{1}{36}, & x \in \{1, 9, 16, 25, 36\}, \\
\frac{1}{18}, & x \in \{2, 3, 5, 8, 10, 15, 18, 20, 24, 30\}, \\
\frac{1}{12}, & x = 4, \\
\frac{1}{9}, & x \in \{6, 12\}. 
\end{cases}
\]

5. Two balls are randomly chosen from a box containing 8 white, 4 black and 2 orange balls. Suppose that we win $2 for each black ball selected and we lose$1 for each white ball selected. Let $X$ be our winning. (a) Find the probability mass function of $X$ when the balls are chosen without replacement; (b) Find the probability mass function of $X$ when the balls are chosen with replacement.
Solution (a)

\[ p(x) = \begin{cases} 
\binom{8}{2} \binom{14}{2} = \frac{28}{91}, & x = -2, \\
\binom{8}{1} \binom{2}{1} \binom{14}{2} = \frac{16}{91}, & x = -1, \\
\binom{2}{2} \binom{14}{2} = \frac{1}{91}, & x = 0, \\
\binom{8}{2} \binom{4}{1} \binom{1}{1} = \frac{32}{91}, & x = 1, \\
\binom{14}{2} \binom{2}{1} \binom{1}{1} = \frac{8}{91}, & x = 2, \\
\binom{2}{2} \binom{14}{2} = \frac{6}{91}, & x = 4 
\end{cases} \]

(b)

\[ p(x) = \begin{cases} 
\frac{8}{13} = \frac{16}{49}, & x = -2, \\
\frac{14}{2} = \frac{8}{49}, & x = -1, \\
\frac{2}{13} = \frac{1}{49}, & x = 0, \\
\frac{2}{13} = \frac{16}{49}, & x = 1, \\
\frac{2}{13} = \frac{4}{49}, & x = 2, \\
\frac{1}{13} = \frac{4}{49}, & x = 4. 
\end{cases} \]