ERRATA for An Introduction to Homological Algebra 2nd Ed.

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Here are all the errata that I know (aside from misspellings). If you have found any errors not listed below, please send them to me at

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Page 3 line 17. change \(da\) to \(dx\) (two times)

Page 4 line 2. change \(\ldots \cup [a,c] \) to \(\ldots \cup [a,b]\)

Page 26 lines \(-4,-3\). should read: For all \(C \in \text{obj}(\mathcal{C})\),

\[(\sigma \tau)_C = \sigma_C \tau_C.\]

Page 30 line 3. change \(t_n\) to \(t_{n-1}\)

Page 38 line 10. \(\ldots \) and \(\alpha \beta(y) = \sum_{x \in G} \alpha(x) \beta(x^{-1}y).\)

line 17. Elements of \(G\), when viewed as elements of \(kG\), multiply as

Page 39 Prop 2.4(i) should read:

\((f + g)_* = f_* + g_*\) for all \(f, g \in \text{Hom}_R(A, B)\).

Page 40 line 17. Definition of additive functor should be placed on page 39.

Page 47 line 12. should read: \(0 \to S/T \to M/T \to M/S \to 0\)

Page 48 line \(-10\). interchange "injection" and "projection"

Page 49 line 14. Change \(\varphi\) to \(\psi\).

Page 49 line \(-10\). Should be: (iv) \(\Rightarrow\) (v)

Page 51 line 7. should read: \(\ker \mathfrak{p} = (T + S')/S'\)

Page 52 In Prop 2.26, should be \(S_i \cap (S_1 + \cdots + \hat{S}_i + \cdots + S_n) = \{0\}\)

Page 53 line \(-18\) should read:

is the map \(\lambda_j: A_j \to B\), defined by \(a_i \mapsto \)

Page 65 Exercise 2.8(i) Note that \(iq\) is a retraction

Page 78 line 9 should be: \(\text{Hom}_S(\sqcup, B)\)

Page 83 line 11. change \(k\)-trilinear to \(k\)-triadditive

Page 85 line \(-16\). should read: \(f: (a,b') \mapsto a \otimes b + E\)

Page 87 line \(-17\). replace line; should read: \(A \otimes_R (\bigoplus_i B_i)\). There is a ho-

Page 89 Move Prop 2.50 to page 88, just before Prop 2.68.
Page 93 line 8. change $\text{Hom}_S(\square \otimes_S B, C)$ to $\text{Hom}_S(\square \otimes_R B, C)$
Page 119 line 7 change $E_i$ to $E_k$
Page 121 line 10 change $rf(1)$ to $rf(a)/a$
Page 125 line 7 change $A$ to $B$
Page 142 line 7 change $A$ to $B$
Page 147 line 3 should read: Let $0 \to A \to$
Page 148 line 17 change middle row to middle column
Page 149 line 5 change $a_i$ to $a_1$
Page 172 line 4 change basis of $V$ to basis of $B$
Page 173 line 9 change Theorem 4.35 to Theorem 4.34
Page 181 line 1 change $g^k$ to $g^{k-1}$
Page 185 line 8 change essential to superfluous
Page 192 line 5 because $s''$ is not a zero-divisor,
Page 195 line 6 change $pR_p$ to $pR_p$
Page 196 line 5 change $S^{-1} \otimes M$ to $S^{-1}R \otimes M$
Page 198 line 8. the composite should have another functor on the right, namely, the change of rings functor $S^{-1}R\text{Mod} \to R\text{Mod}$ induced by the localization map $h: R \to S^{-1}R$
Page 207 line 3 change $f(x)$ to $f(y)$
Page 217 line 1 change $C$ to $B$ (3 times)
Page 218 line 2 change $X$ to $C$
Page 230 lines 1, 2 should read: inverse system in $C$ over $I$
Example 5.16(i): interchange $A$ and $C$ in the diagram
Page 233 line 11 change $\psi_N^M$ to $\psi_M^N$
Page 235 line 5 change $b_n + JM$ to $b_n + J^nM$
Page 237  top diagram  vertical arrows should point down
   line 11  should read: $\varphi^j_i : M_i \rightarrow M_j$

Page 258  line 11  change $\text{Hom}(A, B \otimes C)$ to $\text{Hom}(A \otimes B, C)$

Page 283  line 23  should read
   $$\lim_{\rightarrow U \ni x} \mathcal{P}_1(U) \rightarrow \lim_{\rightarrow U \ni x} \mathcal{P}_2(U) :$$

Page 286  line −9  should read
   because, if $x$ is a closed point, then all the stalks of $x_*A$ are $\{0\}$
   except $(x_*A)_x$, which is $A$.

Page 302  Exercise 5.42  should read
   Give an example of a presheaf of abelian groups on a discrete
   space $X$ which is not a sheaf.

Page 316  line 15  should read $F : A \rightarrow R\text{Mod}$ for some ring $R$.

Page 321  Exercise 5.57  in every abelian category

Page 327  lines 6, 7  change $\pi$ to $p$ (2 times)

Page 333  line 12, 13  should read $= \text{dic'} = \text{id'}c$,

Page 335  line 2  change $\text{Comp}(A)$ to $A$.

Page 336  top diagram  remove subscript * from vertical arrows $f, g, h$
   line 3 below top diagram
   $$f_*\partial\text{cls}(z'') = \text{cls}(fz').$$

Page 337  Change 1c to (1c)*.

Page 343  line 17  change $\partial^*_n$ and $\partial^*_{n-1}$ to $\partial^*_{n+1}$ and $\partial^*_n$

Page 350  line −3 of first paragraph  change $K_0$ to $V$ (twice)
   bottom: Change $\delta_{n+1}$ to $d_{n+1}$ (twice)

Page 359  first diagram: interchange $\partial$ and $\partial'$

Page 360  line −8.  change $H_q$ to $H_n$
   Use Proposition 2.70 to prove the case $n = 1$

Page 361  third diagram  change $\tau_{n-1,K}$ on far right to $T_{n-1}g$

Page 365  line −9.  $B \in \text{obj}(A)$

Page 366  line 3.  change $\text{im } \eta$ to $\text{im } d_0$ and $\text{im } d^{n-1}$ to $\text{im } d^n$

Page 373  line 9.  change $\text{Ext}$ to $\text{ext}$
Prof. A. Azizi found a gap in the proof of Theorem 7.5; on page 408, line 6, why is $H_1(F_A, B) \cong \text{im } \gamma$? He also sent me a corrected version which I have rewritten.

Page 406 Replace the paragraph beginning “We are going to use” by

The following two results will be used in the next proof. The first is a generalization of the First Isomorphism Theorem.

If $f : A \to B$ is a homomorphism and $N \subseteq \ker f$, then $\tilde{f} : A/N \to B$, given by $\tilde{f} : a + N \mapsto fa$, is a well-defined homomorphism with $\ker \tilde{f} = \ker f/N$ and $\text{im } \tilde{f} = \text{im } f$.

The second result is a variation of the Snake Lemma, due to Cartier–Weil.

If $f : A \to B$ and $g : B \to C$ are homomorphisms, then there is an exact sequence

$$0 \to \ker f \to \ker (gf) \to \ker g \to \text{coker } f \to \text{coker } (gf) \to \text{coker } g \to 0.$$  

We sketch a proof. There is a commutative diagram with exact rows

$$
\begin{array}{c}
0 \\
\downarrow f \\
0 \\
\end{array}
\begin{array}{ccccccc}
A & \longrightarrow & A \oplus B & \longrightarrow & B & \longrightarrow & 0 \\
\downarrow h & & \downarrow g & & & & \\
B & \longrightarrow & B \oplus C & \longrightarrow & C & \longrightarrow & 0;
\end{array}
$$

the maps in the rows are the usual injections and projections of direct sums, while $h(a, b) = (fa - b, gb)$. The Snake Lemma gives exactness of

$$0 \to \ker f \to \ker h \to \ker g \to \text{coker } f \to \text{coker } h \to \text{coker } g \to 0,$$

and it is easy to see that $\ker h \cong \ker (gf)$ and $\text{coker } h \cong \text{coker } (gf)$.

Page 407, line 11 through Page 408, line 7 Replace with the following.

For $n = 1$, let $h = d_1 \otimes 1 : F_1 \otimes_R B \to F_0 \otimes_R B$. Since $\text{im } (d_2 \otimes 1) \subseteq \ker (d_1 \otimes 1)$, the generalized First Isomorphism Theorem says that $\tilde{h} : F_1 \otimes_R B/\text{im } (d_2 \otimes 1) \to F_0 \otimes_R B$ has kernel

$$\ker \tilde{h} = \frac{\ker (d_1 \otimes 1)}{\text{im } (d_2 \otimes 1)} = H_1(F_A, B).$$

(1)

Let $Y = \text{im } d_1$, let $i : Y \to F_0$ be the inclusion, and let $d'_1 : F_1 \to Y$ be given by $d'_1(x) = d_1(x)$ [so $d'_1$ differs from $d_1$ only in its target]. Of course, $id'_1 = d_1$.

Define $f = d'_1 \otimes 1 : F_1 \otimes_R B \to Y \otimes_R B$. Note that $d'_1$ is surjective, so that right exactness of $\square \otimes_R B$ gives $f = d'_1 \otimes 1$ surjective; that is, $Y \otimes_R B = (\text{im } d_1) \otimes_R B = \text{im } (d'_1 \otimes 1)$. Since $\text{im } (d_2 \otimes 1) \subseteq \ker (d'_1 \otimes 1)$, the generalized First Isomorphism Theorem gives $\tilde{f} : F_1 \otimes_B/\text{im } (d_2 \otimes 1) \to Y \otimes_R B$ surjective and

$$\ker \tilde{f} = \ker (d'_1 \otimes 1)/\text{im } (d_2 \otimes 1).$$
Let \( g = i \otimes 1 : Y \otimes R B \to F_0 \otimes R B \). The Cartier–Weil variation gives exactness of
\[
\ker \tilde{f} \to \ker(g \tilde{f}) \to \ker g \to \coker \tilde{f}.
\]
We have seen that \( \tilde{f} \) is surjective, so that \( \coker \tilde{f} = \{0\} \). Moreover, exactness of \( F_2 \to F_1 \to Y \to 0 \) gives exactness of
\[
F_2 \otimes R B \xrightarrow{d_2 \otimes 1} F_1 \otimes R B \xrightarrow{d_1 \otimes 1} Y \otimes R B \to 0,
\]
because \( \Box \otimes R B \) is right exact. Hence, \( \text{im}(d_2 \otimes 1) = \ker(d_1' \otimes 1) \), and so \( \ker \tilde{f} = \ker(d_1' \otimes 1) / \text{im}(d_2 \otimes 1) = \{0\} \). Thus, \( \ker(g \tilde{f}) \cong \ker g \). But \( g \tilde{f} = \tilde{h} \), and Eq. (1) gives \( \ker \tilde{h} \cong H_1(F_A, B) \). Therefore,
\[
H_1(F_A, B) \cong \ker g = \ker(i \otimes 1).
\]
Page 487  line −1. Change mod $m$ to mod $m^2$

Page 488  line −6. change $\{f/g \in k[V]\}$ to $\{f/g \in k(V)\}$

Page 489  Lemma 8.59  add hypothesis: $R$ is noetherian

Page 490  Proposition 8.61  add hypothesis: $R$ is noetherian

Page 519  line 8. change $\text{Hom}_G(A, B)$ to $\text{Hom}_Z(A, B)$

Page 560  bottom 3 lines should read

$\text{Hom}_S(Z, A)$ is a (left) $G$-module as follows: if $y \in G$ and $g: ZG \rightarrow A$, define

$$yg: x \mapsto g(y^{-1}x).$$

Page 614  line −10. a subscript $Q$ should be $q$: $\Delta'_p \otimes 1_{B_q}$

Page 629  line −8. change subscript on $(F^{p-1})_n$ to $n-1$

Page 632, 633  interchange the rows in the $2 \times 2$ matrices

Page 667  lines −7, −6

$$\text{Hom}_S(B \otimes_R A, C) \cong \text{Hom}_R(A, \text{Hom}_S(B, C)).$$

If $G = B \otimes_R \square$ and $F = \text{Hom}_S(\square, C)$,

Page 678  line 10. change "isomorphic" to "chain equivalent"

Page 679  line 3. also assume that the complex $Z$ of cycles is flat

Page 692  add references
