

ERRATA for *An Introduction to Homological Algebra* 2nd Ed.

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Here are all the errata that I know (aside from misspellings). If you have found any errors not listed below, please send them to me at

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Page 3 line 17. change da to dx (two times)

Page 4 line 2. change $\dots \cup [a, c]$ to $\dots \cup [a, b]$

Page 26 lines $-4, -3$. should read: For all $C \in \text{obj}(\mathcal{C})$,

$$(\sigma\tau)_C = \sigma_C\tau_C.$$

Page 30 line 3. change t_n to t_{n-1}

Page 38 line 10. ... and $\alpha\beta(y) = \sum_{x \in G} \alpha(x)\beta(x^{-1}y)$.

line 17. Elements of G , when viewed as elements of kG , multiply as

Page 39 Prop 2.4(i) should read:

$$(f + g)_* = f_* + g_* \text{ for all } f, g \in \text{Hom}_R(A, B).$$

Page 40 line 17. Definition of additive functor should be placed on page 39.

Page 47 line 12. should read: $0 \rightarrow S/T \rightarrow M/T \rightarrow M/S \rightarrow 0$

Page 48 line -10 . interchange "injection" and "projection"

Page 49 line 14. Change φ to ψ .

Page 49 line -10 . Should be: (iv) \Rightarrow (v)

Page 51 line 7. should read: $\ker \bar{\rho} = (T + S')/S'$

Page 52 In Prop 2.26, should be $S_i \cap (S_1 + \dots + \widehat{S}_i + \dots + S_n) = \{0\}$

Page 53 line -18 should read:

is the map $\lambda_j: A_j \rightarrow B$, defined by $a_i \mapsto$

Page 65 Exercise 2.8(i) Note that iq is a retraction

Page 78 line 9 should be: $\text{Hom}_S(\square, B)$

Page 83 line 11. change k -trilinear to k -triadditive

Page 85 line -16 . should read: $f: (a, b'') \mapsto a \otimes b + E$

Page 87 line -17 . replace line; should read: $A \otimes_R (\bigoplus_i B_i)$. There is a ho-

Page 89 Move Prop 2.50 to page 88, just before Prop 2.68.

- Page 93 line 8. change $\text{Hom}_S(\square \otimes_S B, C)$ to $\text{Hom}_S(\square \otimes_R B, C)$
line 10 $\text{Hom}_S(B \otimes_R \square, C)$
- Page 119 line 7 change E_i to E_k
- Page 121 line -10 change $rf(1)$ to $rf(a)/a$
- Page 125 line -3 should read: If E is a proper essential
- Page 133 line -12 should be: $= \sum \kappa(m_j) \otimes a_j$
- Page 142 line 7 change A to B
- Page 147 line 3 should read: Let $0 \rightarrow A \rightarrow$
line -17 change middle row to middle column
- Page 148 line -3. change $\{a_i + K : i = 1, \dots, m\}$ to $\{a_j + K : j = 1, \dots, n\}$
- Page 149 line 5 change a_i to a_1
line 9 Lemma 3.68
change second item in inequalities to: $\sum_j a_j \otimes \left(\sum_i r_{ji} h'_i \right)$
- Page 172 line -4 change basis of V to basis of B
- Page 173 line -9 change Theorem 4.35 to Theorem 4.34
- Page 181 line -1 change g^k to g^{k-1}
- Page 185 line -8 change *essential* to *superfluous*
- Page 192 line 5 because s'' is not a zero-divisor,
- Page 195 line 6 change $\mathfrak{p}R\mathfrak{p}$ to $\mathfrak{p}R_{\mathfrak{p}}$
- Page 196 line -5 change $S^{-1} \otimes M$ to $S^{-1}R \otimes M$
- Page 198 line -8. the composite should have another functor on the right, namely, the change of rings functor ${}_{S^{-1}R}\mathbf{Mod} \rightarrow {}_R\mathbf{Mod}$ induced by the localization map $h: R \rightarrow S^{-1}R$
- Page 207 line 3 change $f(x)$ to $f(y)$
- Page 217 line 1 change C to B (3 times)
- Page 218 line 2 change X to C
- Page 230 lines 1,2 should read: **inverse system in \mathcal{C} over I**
Example 5.16(i): interchange A and C in the diagram
line -8 change modules to objects.
- Page 233 line -11 change ψ_N^M to ψ_M^N
line -8 change $\psi_M^D f_M$ to $\psi_D^M f_D$
- Page 235 line 5 change $b_n + JM$ to $b_n + J^n M$

Page 237 top diagram vertical arrows should point down

line 11 should read: $\varphi_j^i: M_i \rightarrow M_j$

Page 258 line 11 change $\text{Hom}(A, B \otimes C)$ to $\text{Hom}(A \otimes B, C)$

Page 283 line 23 should read

$$\lim_{\rightarrow U \ni x} \mathcal{P}_1(U) \rightarrow \lim_{\rightarrow U \ni x} \mathcal{P}_2(U) :$$

Page 286 line -9 should read

because, if x is a closed point, then all the stalks of x_*A are $\{0\}$ except $(x_*A)_x$, which is A .

Page 302 Exercise 5.42 should read

Give an example of a presheaf of abelian groups on a discrete space X which is not a sheaf.

Page 316 line 15 should read $F: \mathcal{A} \rightarrow {}_R\mathbf{Mod}$ for some ring R .

Page 321 Exercise 5.57 in every abelian category

Page 327 lines 6, 7 change π to p (2 times)

Page 333 line 12, 13 should read $= dic' = id'c$,

Page 335 line 2 change $\mathbf{Comp}(\mathcal{A})$ to \mathcal{A} .

Page 336 top diagram remove subscript $*$ from vertical arrows f, g, h

line 3 below top diagram

$$f_*\partial\text{cls}(z'') = \text{cls}(fz').$$

Page 337 Change $1_{\mathbf{C}}$ to $(1_{\mathbf{C}})_*$

Page 343 line 17 change ∂_n^* and ∂_{n-1}^* to ∂_{n+1}^* and ∂_n^*

Page 350 line -3 of first paragraph change K_0 to V (twice)

bottom: Change δ_{n+1} to d_{n+1} (twice)

Page 359 first diagram: interchange ∂ and ∂'

Page 360 line -8. change H_q to H_n

Use Proposition 2.70 to prove the case $n = 1$

Page 361 third diagram change $\tau_{n-1, K}$ on far right to $T_{n-1}g$

Page 365 line -9. $B \in \text{obj}(\mathcal{A})$

Page 366 line 3. change $\text{im } \eta$ to $\text{im } d_0$ and $\text{im } d^{n-1}$ to $\text{im } d^n$

Page 373 line 9 change Ext to ext

Prof. A. Azizi found a gap in the proof of Theorem 7.5; on page 408, line 6, why is $H_1(\mathbf{F}_A, B) \cong \text{im } \gamma$? He also sent me a corrected version which I have rewritten.

Page 406 Replace the paragraph beginning “We are going to use” by

The following two results will be used in the next proof. The first is a generalization of the First Isomorphism Theorem.

If $f: A \rightarrow B$ is a homomorphism and $N \subseteq \ker f$, then $\tilde{f}: A/N \rightarrow B$, given by $\tilde{f}: a + N \mapsto fa$, is a well-defined homomorphism with $\ker \tilde{f} = \ker f/N$ and $\text{im } \tilde{f} = \text{im } f$.

The second result is a variation of the Snake Lemma, due to Cartier–Weil.

If $f: A \rightarrow B$ and $g: B \rightarrow C$ are homomorphisms, then there is an exact sequence

$$0 \rightarrow \ker f \rightarrow \ker(gf) \rightarrow \ker g \rightarrow \text{coker } f \rightarrow \text{coker}(gf) \rightarrow \text{coker } g \rightarrow 0.$$

We sketch a proof. There is a commutative diagram with exact rows

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & A \oplus B & \longrightarrow & B & \longrightarrow & 0 \\ & & \downarrow f & & \downarrow h & & \downarrow g & & \\ 0 & \longrightarrow & B & \longrightarrow & B \oplus C & \longrightarrow & C & \longrightarrow & 0; \end{array}$$

the maps in the rows are the usual injections and projections of direct sums, while $h(a, b) = (fa - b, gb)$. The Snake Lemma gives exactness of

$$0 \rightarrow \ker f \rightarrow \ker h \rightarrow \ker g \rightarrow \text{coker } f \rightarrow \text{coker } h \rightarrow \text{coker } g \rightarrow 0,$$

and it is easy to see that $\ker h \cong \ker(gf)$ and $\text{coker } h \cong \text{coker}(gf)$.

Page 407, line 11 through Page 408, line 7 Replace with the following.

For $n = 1$, let $h = d_1 \otimes 1: F_1 \otimes_R B \rightarrow F_0 \otimes_R B$. Since $\text{im}(d_2 \otimes 1) \subseteq \ker(d_1 \otimes 1)$, the generalized First Isomorphism Theorem says that $\tilde{h}: F_1 \otimes_R B / \text{im}(d_2 \otimes 1) \rightarrow F_0 \otimes_R B$ has kernel

$$\ker \tilde{h} = \frac{\ker(d_1 \otimes 1)}{\text{im}(d_2 \otimes 1)} = H_1(\mathbf{F}_A, B). \quad (1)$$

Let $Y = \text{im } d_1$, let $i: Y \rightarrow F_0$ be the inclusion, and let $d'_1: F_1 \rightarrow Y$ be given by $d'_1: x \mapsto d_1(x)$ [so d'_1 differs from d_1 only in its target]. Of course, $\text{id}'_1 = d_1$.

Define $f = d'_1 \otimes 1: F_1 \otimes_R B \rightarrow Y \otimes_R B$. Note that d'_1 is surjective, so that right exactness of $\square \otimes_R B$ gives $f = d'_1 \otimes 1$ surjective; that is, $Y \otimes_R B = (\text{im } d_1) \otimes_R B = \text{im}(d'_1 \otimes 1)$. Since $\text{im}(d_2 \otimes 1) \subseteq \ker(d'_1 \otimes 1)$, the generalized First Isomorphism Theorem gives $\tilde{f}: F_1 \otimes_R B / \text{im}(d_2 \otimes 1) \rightarrow Y \otimes_R B$ surjective and

$$\ker \tilde{f} = \ker(d'_1 \otimes 1) / \text{im}(d_2 \otimes 1).$$

Let $g = i \otimes 1: Y \otimes_R B \rightarrow F_0 \otimes_R B$. the Cartier–Weil variation gives exactness of

$$\ker \tilde{f} \rightarrow \ker(g\tilde{f}) \rightarrow \ker g \rightarrow \text{coker } \tilde{f}.$$

We have seen that \tilde{f} is surjective, so that $\text{coker } \tilde{f} = \{0\}$. Moreover, exactness of $F_2 \rightarrow F_1 \rightarrow Y \rightarrow 0$ gives exactness of

$$F_2 \otimes_R B \xrightarrow{d_2 \otimes 1} F_1 \otimes_R B \xrightarrow{d_1 \otimes 1} Y \otimes_R B \rightarrow 0,$$

because $\square \otimes_R B$ is right exact. Hence, $\text{im}(d_2 \otimes 1) = \ker(d_1' \otimes 1)$, and so $\ker \tilde{f} = \ker(d_1' \otimes 1)/\text{im}(d_2 \otimes 1) = \{0\}$. Thus, $\ker(g\tilde{f}) \cong \ker g$. But $g\tilde{f} = \tilde{h}$, and Eq. (1) gives $\ker \tilde{h} \cong H_1(\mathbf{F}_A, B)$. Therefore,

$$H_1(\mathbf{F}_A, B) \cong \ker g = \ker(i \otimes 1).$$

Page 411 line –8 change *essential* to *superfluous*

Page 422 line just below diagram. change $GR \cong C$ to $GR \cong A$

Page 427 line –13. change Proposition 6.19 to Exercise 6.19

Page 429, 430 Formula II: change $e(C, A')$ to $\text{Ext}^1(C, A')$

Formula III: change $e(C', A)$ to $\text{Ext}^1(C', A)$

Page 453 line –6. should read

$$\text{Ext}_R^n(A, B) = \frac{\ker d_{n+1}^*}{\text{im} d_n^*},$$

Page 465 line 9. change $\text{fd}(R)$ to $\text{fd}(A)$

Page 466 Exercise 8.5 $0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0$

Page 474 line 1 Change $\text{pd}_R(K^*) \leq 2$ to $\text{pd}_R(M^*) \leq 2$

Page 475 line 5. Change ${}_R\mathbf{Mod}$ to \mathbf{Mod}_R

Page 475 line –1. $a \otimes a' \otimes m \mapsto aa' \otimes m$

Page 477 line –12. change $(d_2, p_2, 0)$ to $(d_2 p_2, 0)$

Page 481 line 12. delete "left" two times

Page 482 lines 16–18 Should read:

$f(x) \in \mathfrak{p} \subseteq R[x]$ of least degree, and consider the exact sequence $0 \rightarrow (f) \rightarrow \mathfrak{p} \rightarrow \mathfrak{p}/(f) \rightarrow 0$. Now $(f) \cong R[x]$, since $R[x]$ is a domain, and $\text{ann}(\mathfrak{p}/(f)) \neq \{0\}$: if $Q = \text{Frac}(R)$, then $\mathfrak{p}Q[x]$ is generated by f , so that if $g \in \mathfrak{p}$, there is $c \in R$ with $cg \in Q[x]$.

Page 486 line –5. $\text{pd}(M) \leq n$

Page 487 line –2. Change $y_i + \mathfrak{m}$ to $y_i + (x)$.

Page 487 line -1. Change mod \mathfrak{m} to mod \mathfrak{m}^2

Page 488 line -6. change $\{f/g \in k[V]\}$ to $\{f/g \in k(V)\}$

Page 489 Lemma 8.59 add hypothesis: R is noetherian

Page 490 Proposition 8.61 add hypothesis: R is noetherian

Page 519 line 8. change $\text{Hom}_G(A, B)$ to $\text{Hom}_{\mathbb{Z}}(A, B)$

Page 560 bottom 3 lines should read

$\text{Hom}_S(\mathbb{Z}, A)$ is a (left) G -module as follows: if $y \in G$ and $g: \mathbb{Z}G \rightarrow A$, define

$$yg: x \mapsto g(y^{-1}x).$$

Page 614 line -10. a subscript Q should be q : $\Delta'_P \otimes 1_{B_q}$

Page 629 line -8. change subscript on $(F^{p-1})_n$ to $n-1$

Page 632, 633 interchange the rows in the 2×2 matrices

Page 667 lines -7, -6

$$\text{Hom}_S(B \otimes_R A, C) \cong \text{Hom}_R(A, \text{Hom}_S(B, C)).$$

If $G = B \otimes_R \square$ and $F = \text{Hom}_S(\square, C)$,

Page 678 line 10. change "isomorphic" to "chain equivalent"

Page 679 line 3. also assume that the complex \mathbf{Z} of cycles is flat

Page 692 add references

Neukirch, J., Schmidt, A., and Wingberg, K., *Cohomology of Number Fields*, Grundlehren der mathematischen Wissenschaften 323, 2d Ed., Springer, 2008.

Northcott, D. G., *A First Course of Homological Algebra*, Cambridge University Press, 1973.