Advanced Modern Algebra; Revised Printing

Errata: August 8, 2005

All the errata that were found in the first printing were, in fact, corrected in the revised (second printing), published in August, 2003. Alas, more errata have since been found, including one that arose when I foolishly changed a proof (on page 355) that was correct in the first printing! Here is the list, with the hope that the number of undiscovered errors is very close to zero. I thank Vincenzo Acciaro, Dan Grayson, Frank Grosshans, Jerry Janusz, Nick Loehr, Marko Moisio, Mark Shimozono, Tyler Smith, Carol Wood, and Mirroslav Yotov for telling me about errors they had found.

Page 13 line -1. Add part (ii) of Exercise 1.26.

(ii) Give an example in which $[a, b, c](a, b, c) \neq abc$.

Page 21. Interchange Proposition 1.38 and Corollary 1.39; rename Corollary 1.39 as Proposition 1.38 and rename Proposition 1.38 as Proposition 1.39.

Page 42 line 4. Should read

The right side is the value of the composite $\beta\gamma\delta$, where

Page 42 line 13. Change "composite" to "composition"

Page 43 line 6. Change "contradicts" to "contradict"

Page 79 line -3 Change "Corollary 2.62(i)" to "Proposition 2.62(i)"

Page 85 line 19. Change "choice" to "choices"

Page 86 line -5.

We point out that any infinite cyclic group is isomorphic ...

Page 93 line 7. Add

where $p_1, \ldots, p_t$ are distinct primes,

Page 94 line -5. Add

This theorem may be false if G is not abelian. The group $\mathbf{Q}$ of quaternions is a nonabelian group of order 8 having exactly one (cyclic) subgroup of order 2.

Page 104 line -6 Change "G" to "$G \neq \{1\}$"

Page 105 line 18.

$$= (a_i, a_{i+1}, \ldots, a_{p-1}, a_0, a_1, \ldots, a_{i-1}).$$

Page 105 line 20.

$$a_i a_{i+1} \cdots a_{p-1} a_0 a_1 \cdots a_{i-1} = (a_0 a_1 \cdots a_{i-1})^{-1} (a_0 a_1 \cdots a_{p-1})(a_0 a_1 \cdots a_{i-1}).$$

Page 106 line 16.

A group G is called ***simple*** if $G \neq \{1\}$ and it has no ...

Page 109 line 1 Change "*Let G act*" to "*Let G be a finite group acting*"

Page 118 line 15. Add

where $na = a + \cdots + a$ is the sum of a with itself n times

Page 122 line 6 Change "a commutative" to "a nonzero commutative"

Page 142 Exercise 3.34 Assume that $1 + 1 \neq 0$ in k.

Page 149 Add a first part to Exercise 3.43.

(i) Let R and S be commutative rings, and let $\varphi \colon R \to S$ be a ring homomorphism. If $s \in S$, prove that there exists a unique ring homomorphism $\widetilde{\varphi} \colon R[x] \to S$ with $\widetilde{\varphi}(r) = \varphi(r)$ for all $r \in R$ and with $\widetilde{\varphi}(x) = s$.

Page 206 line -4. Delete "then $k(u)$"

Page 207 Delete lines 11 through 17. Replace by following.

For every field k and every $m \geq 1$, we show that the polynomial $f(x) = x^m - 1 \in k[x]$ is solvable by radicals. Recall that the set Γ_m of all mth roots of unity in a splitting field E/k of $f(x)$ is a cyclic group, say, with generator ζ. Note that $|\Gamma_m| = m$ unless k has characteristic $p > 0$ and $p \mid m$, in which case $|\Gamma_m| = m'$, where $m = p^e m'$ and $p \nmid m'$. Now $E = k(\zeta)$, so that E is a pure extension of k, and hence E/k is a radical extension. Therefore, $f(x) = x^m - 1$ is solvable by radicals.

Page 217 Exercise 4.4. Assume that $f(x)$ is monic.

Page 218 Exercise 4.11 Should read "E/k is a splitting field of $f(x)$."

(ii) Let E/k be a finite extension. Prove that E/k is a splitting field of some polynomial in $k[x]$ if and only if every irreducible polynomial $p(x) \in k[x]$ having a root in E splits in $E[x]$. (Compare with Theorem 4.34 which uses a separability hypothesis.)

Page 224 line 18 Should read "... containing k and the elementary"

Pages 247, 248 Exercise 4.23. Rewrite without parts.

4.23. Let E/k be a Galois extension with $[E : k] = n$ and with cyclic Galois group $G = \mathrm{Gal}(E/k)$, say, $G = \langle \sigma \rangle$. Define $\tau = \sigma - 1_E$, and prove that $\ker \tau = \ker T$, where $T \colon E \to E$ is the trace. Conclude, in this case, that the ***Trace Theorem*** is true:

$$\ker T = \{a \in E : a = \sigma(u) - u \text{ for some } u \in E\}.$$

Hint. Show that $\ker \tau = k$, so that $\dim(\mathrm{im}\, \tau) = n - 1 = \dim(\ker T)$.

Page 254 line 13 Should begin "so that if $G = \mathbb{Z}^n$, then $|G/2G| = 2^n$."

Page 255 line -9 Should end "claim that the map $g \colon A \to \mathbb{Z}^n$ is an"

Page 267 line -7 Add: "For a cyclic group $G = \langle a \rangle$ of order m and ..."

Page 296 line -6 Change "prime p" to "prime p dividing $|\,\mathrm{SL}(2, \mathbb{F}_5)|$"

Page 300 line 5 Add following footnote.

An alternative method of constructing a free group on a set X uses the ***van der Waerden trick***: let Ω be the set of all the reduced words on X and show that a certain subgroup of the symmetric group S_Ω is free with basis X (see Rotman, *An Introduction to the Theory of Groups*, pages 344–345).

Page 322 line -12 . Replace "Theorem 6.101" by "Theorem 6.102"

Page 325 Exercise 6.10(ii). Replace "If $I = (2)$," by "If $I = (2) = J$,"

Page 338 In proof of Theorem 6.34, change $\mathbb{Z}$ to R, $\mathbb{F}_p$ to $R/(p)$, where (p) is a nonzero prime ideal, and $\mathbb{Q}$ to $\mathrm{Frac}(R)$.

Page 343 Replace the first three lines of the proof of the Hilbert basis theorem with following.

Assume that I is an ideal in $R[x]$ which is not finitely generated; of course, $I \neq \{0\}$. Define $f_0(x)$ to be a polynomial in I of minimal degree and define, inductively, $f_{n+1}(x)$ to be a polynomial of minimal degree in $I - (f_0, \ldots, f_n)$. Note that $f_n(x)$ exists for all $n \geq 0$; if $I - (f_0, \ldots, f_n)$ were empty, then I would be finitely generated. It is clear that

Page 353 line -14 Replace "Corollary 3.90 on page 170" by "Proposition 6.54(ii)"

Page 355 Change lines 17 through 22, as follows.

It turns out that K is algebraically closed, but a proof is tricky [see I. M. Isaacs "Roots of Polynomials in Algebraic Extensions of Fields," *American Mathematical Monthly* 87 (1980), 543–544]. We avoid this proof by the following construction. Define $k_1 = K$ and iterate: Construct k_{n+1} from k_n in the same way K is constructed from k. There is a tower of fields

$$k = k_0 \subseteq k_1 \subseteq \cdots \subseteq k_n \subseteq k_{n+1} \subseteq \cdots$$

with each extension k_{n+1}/k_n algebraic and with every nonconstant polynomial in $k_n[x]$ having a root in k_{n+1}. By Lemma 6.56(ii), $\Omega = \bigcup_n k_n$ is an algebraic extension of k. We claim that Ω is algebraically closed. If $g(x) = \sum_{i=0}^{m} \omega_i x^i \in \Omega[x]$ is a nonconstant polynomial, then it has only finitely many coefficients $\omega_0, \ldots, \omega_m$, and so there is some k_q that contains them all. It follows that $g(x) \in k_q[x]$ and so $g(x)$ has a root in $k_{q+1} \subseteq \Omega$, as desired. Therefore, Ω is an algebraic closure of k. •

Page 359 Replace lines -15 to -1 by:

Each coefficient $\beta_\ell \in B \subseteq k(x)$ is a rational function, say, $\beta_\ell = g_\ell(x)/h_\ell(x)$, where $g_\ell(x), h_\ell(x) \in k[x]$; we may assume that each g_ℓ/h_ℓ is in lowest terms; that is, $(g_\ell, h_\ell) = 1$. Denote $\mathrm{lcm}\{h_0, \ldots, h_{n-1}\}$ by $f(x)$. Thus, there are $u_\ell(x) \in k[x]$, for all ℓ, with $f(x) = u_\ell(x) h_\ell(x)$; moreover, $\gcd\{u_0, \ldots, u_{n-1}\} = 1$. We claim that

$$i(x, y) = f(x)\,\mathrm{irr}(x, B) = f(x)y^n + u_{n-1}g_{n-1}y^{n-1} + \cdots + u_0 g_0 \in k[x][y]$$

is a primitive polynomial (of course, $k[x][y] = k[x, y]$, but we wish to view it as polynomials in y with coefficients in $k[x]$). If $i(x, y)$ is not primitive, then there is an irreducible $p(x) \in k[x]$ dividing $f(x)$ and each $u_\ell g_\ell$. Now $f = u_\ell h_\ell$ for every ℓ. If $p \nmid u_\ell$ for some ℓ, then $p \mid h_\ell$, by Euclid's lemma in $k[x]$. It follows that $p \nmid g_\ell$, because $(g_\ell, h_\ell) = 1$.But $p \mid u_\ell g_\ell$, the ℓth coefficient of $i(x, y)$, so that Euclid's lemma gives $p \mid u_\ell$, a contradiction. We conclude that $p \mid u_\ell$ for all ℓ, which contradicts $\gcd\{u_{n-1}, \ldots, u_0\} = 1$. By Lemma 6.24(i), $f(x)^{-1}$ is the content of $\operatorname{irr}(x, B)$.

If we denote the highest exponent of y occurring in a polynomial $a(x, y)$ by $\deg_y(a)$, then $n = \deg_y(i)$; let $m = \deg_x(i)$. Since $i(x, y) = f(x)y^n + \sum_{\ell=0}^{n-1} f(x)\beta_\ell y^\ell$, we have $m = \max_\ell\{\deg(f), \deg(f\beta_\ell)\}$. Now $h_\ell(x) \mid f(x)$ for all ℓ, so that $\deg(h_\ell) \le \deg(f) \le m$ [because $f(x)$ is one of the coefficients of $i(x, y)$]. Also, $\deg(g_\ell) \le \deg(u_\ell g_\ell) = \deg(f\beta_\ell) \le m$, for $f\beta_\ell = \operatorname{lcm}\{h_0, \ldots, h_{n-1}\}g_\ell/h_\ell = (\operatorname{lcm}\{h_0, \ldots, h_{n-1}\}/h_\ell)g_\ell \in k[x]$. We conclude that $\deg(g_\ell) \le m$ and $\deg(h_\ell) \le m$.

Page 375 Exercise 6.55(i) is false. Give a counterexample.

Pages 385-386 Interchange Theorems 6.101 and 6.102, and change the proof of (the present) 6.101 as follows:

Proof. By Theorem 6.101, $\operatorname{Id}(\operatorname{Var}(M)) = \sqrt{M} = M$, because M is a maximal, hence prime, ideal. Since M is a proper ideal, we have $\operatorname{Var}(M) \ne \varnothing$, by Theorem 6.100; that is, there is $a = (a_1, \ldots, a_n) \in k^n$ with $f(a) = 0$ for all $f \in M$. Hence, $\{a\} \subseteq \operatorname{Var}(M) = \{b \in k^n : f(b) = 0 \text{ for all } f \in M\}$, and Proposition 6.92 gives $M = \operatorname{Id}(\operatorname{Var}(M)) \subseteq \operatorname{Id}(\{a\})$. Since $\operatorname{Id}(\{a\})$ does not contain any nonzero constant, it is a proper ideal, and so maximality of M gives $M = \operatorname{Id}(\{a\}) = \{f(X) \in k[X] : f(a) = 0\}$. For each i, define $f_i(X) = x_i - a_i$. Now $f_i(a) = 0$, so that $(f_1, \ldots, f_n) \subseteq \operatorname{Id}(\{a\})$. But $(x_1 - a_1, \ldots, x_n - a_n)$ is a maximal ideal, by Exercise 6.6(i), so that $(f_1, \ldots, f_n) = (x_1 - a_1, \ldots, x_n - a_n) = \operatorname{Id}(\{a\}) = M$. •

Page 390 line 6. Should be: "If k is algebraically closed, then every radical ideal ..."

Page 408 line −12. Should read:

select smallest i with $\operatorname{LT}(g_i) \mid c_\beta X^\beta$ for β maximal such that

Pages 411, 412 Change the first paragraph of the proof of Proposition 6.129 as follows.

Proof. Assume there is some permutation $\sigma \in S_m$ and some $f(X) = \sum_\alpha c_\alpha X^\alpha \in I$ whose remainder mod G_σ is not 0. Arrange the multidegrees of its terms in descending order, $\alpha_1 \succ \cdots \succ \alpha_p$, and, as in the proof of Lemma 6.123, define multiword$(f) = \alpha_1 \cdots \alpha_p \in \mathcal{W}^+(\mathbb{N}^n)$. Among all such polynomials f, choose one minimal in the well-ordered set $\mathcal{W}^+(\mathbb{N}^n)$. Since $\{g_1, \ldots, g_m\}$ is a Gröbner basis, $\operatorname{LT}(g_i) \mid \operatorname{LT}(f)$ for some i. Apply the division algorithm to obtain a new polynomial, say, h, and note that $h \in I$. Since multidegree$(h) \prec$ multidegree(f), by Proposition 6.126, h reduces to 0 mod G_σ. Therefore, f also reduces to 0, which is a contradiction.

Page 428 line 11. Assume that R is a commutative ring.

Page 444 line 4. Replace "κ_y^x" by "$\{\kappa_y^x\}$"

Page 446 line 1. Delete last part of sentence; should read:

for which $pi = 1_S$.

Page 451 line −7. Replace "$m = \sum_{i \in I} \alpha_i(a)$," by "$m = \sum_{i \in I} \alpha_i(a_i)$,"

Page 451 line −8. Replace "$\alpha_i(a)$" by "$\alpha_i(a_i)$"

Page 452 lines 3 and 5 and first diagram: replace "$\bigsqcup_{i \in I} A_i$" by "C"

Page 452 line 7. Should read:

Should it exist, a coproduct is denoted by $\bigsqcup_{i \in I} A_i$, and it is unique to equivalence.

Page 452 line −13. Replace "$\alpha_i(a)$" by "$\alpha_i(a_i)$'

Page 453 lines 6 and 8 and first diagram: Replace "$\prod_{i \in I} A_i$" by "C"

Page 453 line 10. Add sentence:

Should it exist, a product is denoted by $\prod_{i \in I} A_i$, and it is unique to equivalence.

Page 453 line 19. Should read "$p_i \psi(x) = f_i(x)$"

Page 460 The hint for Exercise 7.32(ii) should read:

Remark. In **Sets**, $(g_i, g_2) = (g_1 \sqcap g_2)\Delta_X$, where $\Delta_X \colon X \to X \times X$ is the diagonal $x \mapsto (x, x)$.

Page 462 lines −12 to −10. Change "$1_A \colon A \to A$" to "$1_B \colon B \to B$" and change "1_A" to "1_B" several times.

Page 463 lines 12, 13. Should read:

(v) If $\mathcal{C} =$ **Groups**, define the ***forgetful functor*** $U \colon$ **Groups** $\to$ **Sets** as follows: $U(G)$ is the "underlying" set of a group G and $U(f)$ is a homomorphism f regarded as a mere function.

Page 471 line −3. Make the statement necessary and sufficient

Proposition 7.49. *Let F be an R-module generated by a subset B. Then F is (isomorphic to) a free R-module with basis B if and only if, for every R-module M and every function $\gamma \colon B \to M$, there exists a unique R-map $g \colon F \to M$ with $g(b) = \gamma(b)$ for all $b \in B$.*

Page 472 New proof of Proposition 7.49.

Proof. Every element $v \in F$ has a unique expression of the form

$$v = \sum_{b \in B} r_b b,$$

where $r_b \in R$ and almost all $r_b = 0$. Define $g \colon F \to M$ by $g(v) = \sum_{b \in B} r_b \gamma(b)$.

Conversely, if we define $A_b = Rb$, the free R-module with basis $\{b\}$, then the condition says that F is a coproduct of $\{A_b : b \in B\}$. In more detail, define injections α_b mapping $r_b b$ to the "vector" having $r_b b$ in the bth coordinate and 0's elsewhere. As for any coproduct, there is a unique map $\theta \colon F \to M$ with $\theta\alpha_b(b) = \gamma(b)$. The maps θ and g agree on each element of the basis B, so that $\theta = g$. By Proposition 7.30, $F \cong \sum_{b \in B} A_b = \sum_{b \in B} Rb$, and so F is isomorphic to a free R-module with basis B. •

Page 486 line 8. Change "By the Proposition" to "By Corollary 7.73"

Page 488 Add an extra part to Exercise 7.54.

If R is a domain, prove that torsion-free divisible R-modules are injective.

Page 492 line 9. Change "$\mathcal{F}(\mathcal{C})|$" to "$\mathcal{F}(\mathcal{C})$"

Page 492 line 10. Change "$|\mathrm{obj}(\mathcal{C})$" to "$|\mathrm{obj}(\mathcal{C})|$"

Page 496 line -1. Delete "a contradiction."

Page 502 line -5. Should read

satisfies the condition $a_n + J^n M = \psi_n^m(a_m + J^m M) = a_m + J^n M$ for all $m \geq n$, so that

Page 503 line 3. Change "and conversely." to

using subsequences, we can see that the converse is essentially true.

Page 514 lines -15, -3 . Change "underlying" to "forgetful"

Page 528 line -6. Change "Proposition 7.24" to "Proposition 7.23"

Page 529 line 1. Should read:

Definition. A ring R is an ordered triple (R, α, μ), where $\alpha\colon R \times R \to R$ is addition and $\mu\colon R \times R \to R$ is multiplication, satisfying certain axioms. Two rings (R, α, μ) and (R', α', μ') are ***equal*** if $R = R'$, $\alpha = \alpha'$, and $\mu = \mu'$. Define the ***opposite ring***

Page 533 Exercise 8.19(ii). Change "$r \in R$" to "$r \in \Delta$"

Page 546 line 11. Should read:

Recall that A^m is the set of all sums of products of the form ...

Page 546 line 11. The proof of Corollary 8.35(i) in the text is correct, but the following proof is nicer.

If $x \in J(R)$, then $xM = \{0\}$ for every simple left R-module M, by Proposition 8.31. But $M \cong R/I$ for some maximal left ideal I; that is, $x \in \mathrm{ann}(R/I)$. Thus, $x \in \bigcap_{\substack{I=\text{maximal}\\ \text{left ideal}}} \mathrm{ann}(R/I)$.

For the reverse inclusion, if $x \in \bigcap_{\substack{I=\text{maximal}\\ \text{left ideal}}} \mathrm{ann}(R/I)$, then $xM = \{0\}$ for every left R-module of the form R/I for some maximal left ideal I. But every simple left R-module M has this form. Therefore, $x \in J(R)$.

Page 549 Exercise 8.27(iii) Change "every semisimple ring" to "every ring which is a direct sum of minimal left ideals"

Page 549 Exercise 8.34; hint: define $r'f : r \mapsto f(rr')$

Page 563-564 line 8. Replace the proof of Proposition 8.59.

Proof. (***Janusz***) Since R is left artinian, it contains a minimal left ideal, say, L; of course, L is a simple left R-module. For each $a \in R$, the function $f_a \colon L \to R$, defined by $f_a(x) = xa$, is a map of left R-modules: if $r \in R$, then

$$f_a(rx) = (rx)a = r(xa) = rf_a(x).$$

Now im $f_a = La$, while L being a simple module forces ker $f_a = L$ or ker $f_a = \{0\}$. In the first case, we have $La = \{0\}$; in the second case, we have $L \cong La$. Thus, La is either $\{0\}$ or a simple module.

Consider the sum $I = \langle \bigcup_{a \in R} La \rangle \subseteq R$. Plainly, I is a left ideal; it is a right ideal as well, for if $b \in R$ and $La \subseteq I$, then $(La)b = L(ab) \subseteq I$. Since R is a simple ring, the nonzero two-sided ideal I must equal R. We claim that R is a sum of only finitely many La's. As any element of R, the unit 1 lies in some finite sum of La's; say, $1 \in Le_1 + \cdots + Le_n$. If $b \in R$, then $b = b1 \in b(Le_1 + \cdots + Le_n) \subseteq Le_1 + \cdots + Le_n$ (because $Le_1 + \cdots + Le_n$ is a left ideal). Hence, $R = Le_1 + \cdots + Le_n$.

To prove that R is semisimple, it remains to show that it is a *direct sum* of simple submodules. Choose n minimal such that $R = Le_1 + \cdots + Le_n$; we claim that $R = Le_1 \oplus \cdots \oplus Le_n$. By Proposition 7.19, it suffices to show that

$$Le_i \cap \Big(\sum_{j \neq i} Le_j\Big) = \{0\}$$

for all i. If this intersection is not $\{0\}$, then simplicity of Le_i says that $Le_i \cap (\sum_{j \neq i} Le_j) = Le_i$; that is, $Le_i \subseteq \sum_{j \neq i} Le_j$, and this contradicts the minimal choice of n. Therefore, R is a semisimple ring. •

Page 566 line −9. Change "$\dim_\Delta(M)$" to "$\dim_\Delta(M)/n$"

Page 566 line −7. Change "$\dim_\Delta(M) = d = \dim_\Delta(N)$" to "$\dim_\Delta(M) = nd = \dim_\Delta(N)$"

Page 608 lines −6 to −1. Replace with the following, which I hope is clearer.

The algebra $\mathbb{C}G = B_1 \oplus \cdots \oplus B_r$, where each B_i is not only a 2-sided ideal in R, it is "almost" a subring of R (its unit is different from the unit in $\mathbb{C}G$). Define a $\mathbb{C}$-linear transformation $F \colon \mathbb{C}G \to \mathbb{C}G$ by

$$F \colon (b_1, \ldots, b_r) \mapsto (\tilde{\lambda}_1(b_1), \ldots, \tilde{\lambda}_r(b_r)).$$

To see that F is a $\mathbb{C}$-algebra map, it suffices to prove that it preserves multiplication. It has already been shown that

$$F(b_i b_i') = \tilde{\lambda}_i(b_i b_i') = \tilde{\lambda}_i(b_i)\tilde{\lambda}_i(b_i') = F(b_i)F(b_i')$$

whenever $b_i, b_i' \in B_i$. But if $b_i \in B_i$ and $b_j \in B_j$ for $i \neq j$, then $b_i b_j = 0$ (because the Bs are ideals and $\mathbb{C}G$ is their direct sum). On the other hand, $F(b_i) \in B_i$ and $F(b_j) \in B_j$, so that $F(b_i)F(b_j) = 0$, too. That is, $F(b_i b_j) = 0 = F(b_i)F(b_j)$, and so F is an algebra map.

Page 609 lines -15, -14. Change "$V^{\langle g \rangle}$" to "$V^{\langle \sigma' \rangle}$"

Page 613 line 13. Change "every class function is a class sum" to "every class sum is a class function"

Page 614 lines 12–14 . The proof of (3) is correct, but we have already proved it another way, in Lemma 8.126, by observing that $\chi_i(1)$ is the trace of the $n_i \times n_i$ identity matrix.

Page 621 line -5. Change "$|\theta(g)| = \mathrm{tr}(I) = \theta(1)$" to "$\theta(g) = \mathrm{tr}(I) = \theta(1)$"

Page 621 line -4. Change "$\left|\sum_{j=1}^{d} \epsilon_j\right| = d$" to "$\sum_{j=1}^{d} \epsilon_j = d$"

Page 622 line 8. Change "$\theta(g) = \sum_j m_j \chi_j(g) \neq \theta(1)$." to

$$\text{“}|\theta(g| \le \sum_j m_j |\chi_j(g)| < \sum_j \chi_j(1) = 1,$$

which implies that $\theta(g) \neq \theta(1)$, a contradiction."

Page 622 line 12. Change "By part (iii)" to "By part (ii)."

Page 623 line 1. Change "$\mathbb{C}G$-module" to "$\mathbb{C}(G/H)$-module"

Page 648 line -9. Now $D \cong S$ is finitely generated, for S is a direct summand, hence an image, of M.

Page 659 line -3. Should read "$\big(\varphi\big(e^{a2\pi i m_p/p^{n_p}}\big)\big)$"

Page 677 line 1. Should read

It follows that Jordan blocks also correspond to polynomials (just as companion matrices do); in

Page 680 line 8. Delete "the coefficient of"

Page 681 Exercise 9.38 Add the remark that if k is a perfect field, then the Jordan decomposition of a matrix is unique.

Page 683 line 9. Change "Proposition 3.101" to "Corollary 3.101"

Page 688 line -6. Change "every entry of Δ'" to "every entry of Δ"

Page 698 line -4. Change "$= \sum_j b_i f(e_j, e_i)$. In matrix" to "$= \sum_i b_i f(e_j, e_i)$. In matrix"

Page 698 line -3. Change "$B^t A = 0$" to "$b^t A = 0$"

Page 700 line -13. Change $V = W \oplus W^{\perp}$ to $V = W \oplus W^{\perp R} = W \oplus W^{\perp L}$

Page 700 line -1. Change "*Every symmetric matrix A*" to "*Every nonsingular symmetric matrix A*"

Page 726 line 1. Delete "identity" at the end of line

Page 731 lines 16, 26, 28. Change "$\mathrm{Mat}_{nr}(A)$" to "$\mathrm{Mat}_{nr}(k)$"

Page 737 line 4. Change "$\mathrm{Mat}_r(\Delta)$" to "$\mathrm{Mat}_r(\Delta^{\mathrm{op}})$"

Page 738 line 2. Change "*is central simple*" to "*is a central simple k-algebra*"

Page 755 Exercise 9.91. The right side should be $(U/U') \otimes (V/V')$; change the hint accordingly.

Page 788 line 17. Note that we are not assuming that the kernel K is abelian.

Page 794 Exercise 10.12. Change the conclusion of part (ii) to read:

prove that $\varphi\colon G \to G$, given by $\varphi\colon a \mapsto (sp+2)g$, are the only automorphisms with $\varphi(pg) = 2pg$.

Page 804 line −5. Change "computational lemma" to "computational result"

Page 807 lines −3, 2. Should read:

that φ is conjugation by $a_0 \in K$ says that $\varphi(a + \ell x) =$

Page 807 line −2. Change every occurrence of "b" to "a_0"

Page 818 line 14. Change "0" to "$\{0\}$"

Page 827 Exercise 10.24. Change "$m \le n$" to "$n \le m$"

Page 829 line 1. line should end "$\varinjlim \mathbf{C}^i_\bullet$ exists in ${}_R\mathbf{Comp}$."

Page 829 Exercise 10.34. Assume that k is a PID.

Page 832 line −7. Change "*exists a chain map* $\check{f}\colon \mathbf{P}_A \to \mathbf{P}'_{A'}$" to "*exists a chain map* $\check{f}\colon \mathbf{P}_\bullet \to \mathbf{P}'_\bullet$"

Page 833 lines −8 to −4. Replace by

$$h_n - \check{f}_n = d'_{n+1}s_n + s_{n-1}d_n.$$

Let us now begin the induction. First, define $\check{f}_{-1} = f = h_{-1}$. If we define $s_{-1} = 0 = s_{-2}$, then

$$h_{-1} - \check{f}_{-1} = f - f = 0 = d'_0 s_{-1} + s_{-2}d_{-1}$$

for any choice of d'_0 and d_{-1}; define $d'_0 = \varepsilon'$ and $d_{-1} = 0$.

Page 835 lines 10, 11. Should read:

Fill in a chain map $\check{f}$ over f, delete A and A', apply T to this diagram, and then take the map induced by $T\check{f}$ in homology.

Page 835 line 14. Should read $\mu_r\colon a \mapsto ra$

Page 836 line −8. Change "Tor_R" to "Tor^R_n"

Page 837 line −3. Change "$1_{T\widetilde{\mathbb{P}}_A}$." to "$1_{T\mathbb{P}_A}$"

Page 838 lines 10 and 12. Change "$\widetilde{\mathbb{P}}_A$" to "$\widetilde{\mathbb{Q}}_B$"

Page 840 line 2. Change "(x, x'')" to "(x', x'')"

Page 840 line 9. Add

and ε is surjective by the Five Lemma, Exercise 8.52 on page 604.

Page 841 line 6. Change "seqeunces" to "sequences"

Page 841 line 9. Change "$L_nTA \xrightarrow{(Ti)_*}$" to "$L_nTA \xrightarrow{(Tj)_*}$"

Page 841 line 13. Change

$L_nTA \xrightarrow{\tau_A^{-1}(Ti)_*} L_nTA \xrightarrow{\tau_A(Tq)_*}$ to $L_nTA \xrightarrow{\tau_A^{-1}(Tj)_*} L_nTA \xrightarrow{(Tq)_*\tau_A}$

Page 841 lines 16 − 22. Should be

It remains to show that $\tau_A^{-1}(Tj)_* = L_nT(i) = T(j)_*$ (remember that $j = \check{i}$, a chain map over i) and $(Tq)_*\tau_A = L_nT(p)$. Now $\tau_A^{-1} = (T\kappa)_*$, where $\kappa : \widetilde{\mathbf{P}}_A \to \mathbf{P}_A$ is a chain map over 1_A, and so

$$\tau_A^{-1}(Tj)_* = (T\kappa)_*(Tj)_* = (T\kappa Tj)_* = \big(T(\kappa j)\big)_*.$$

Both κj and j are chain maps $\widetilde{\mathbf{P}}_A \to \mathbf{P}_A$ over 1_A, so they are homotopic, by the comparison theorem. Hence, $T(\kappa j)$ and Tj are homotopic, and so they induce the same map in homology: $(T(\kappa j))_* = (Tj)_* = L_nT(i)$. Therefore, $\tau_A^{-1}(Tj)_* = L_nT(i)$. We prove $(Tq)_*\tau_A = L_nT(p)$ in the same way. •

Page 841 line −1. Should read:

If $A \to B \to C \to 0$ is exact, then $L_0A \to L_0B \to L_0C \to 0$ is exact. •

Page 842 lines 10 − 13. Should read:

$$L_0TA = \operatorname{coker} T(d_1).$$

But right exactness of T gives an exact sequence

$$TP_1 \xrightarrow{Td_1} TP_0 \xrightarrow{T\varepsilon} TA \to 0.$$

Now $T\varepsilon$ induces an isomorphism $\sigma_A \colon \operatorname{coker} T(d_1) \to TA$, by the first isomorphism

Page 843 line 12. Change "Theorem 10.44" to "Lemma 10.53"

Page 844 last two lines of Step 3. Change targets of η and τ from B to C

Pages 845, 846 Change Step 5 as follows

Step 5. It remains to choose γ so that the square with vertices P, Q, C, and A commutes; that is, we want $g\varepsilon = \eta F$. Evaluating each side leads to the equation

$$gi\varepsilon'x' + g\sigma x'' = j\eta'F'x' + j\eta'\gamma x'' + \tau F''x''.$$

Now $gi\varepsilon' = jf\varepsilon' = j\eta' F'$ (because F' is the 0th term in the chain map $\check{f}$ over f), and so it suffices to find γ so that

$$j\eta'\gamma = g\sigma - \tau F''.$$

Consider the diagram with exact row:

$$\begin{array}{ccccc} & & P'' & & \\ & & \big\downarrow{\scriptstyle g\sigma-\tau F''} & & \\ Q' & \xrightarrow{j\eta'} & C & \xrightarrow{q} & C'' \end{array}$$

Now $\operatorname{im}(g\sigma - \tau F'') \subseteq \operatorname{im} j\eta' = \ker q$, for

$$qg\sigma - q\tau F'' = hp\sigma - \eta'' F'' = h\varepsilon'' - \eta'' F'' = 0.$$

Since P'' is projective, there exists a map $\gamma\colon P'' \to Q'$ making the diagram commute.

Page 846 line 6. Delete an unnecessary "the"

Pages 845, 846 16 line -7 Change "$\to \operatorname{Hom}(B, E'^n)$ to "$\to \operatorname{Hom}(B, E^{n+1})$

Page 849 line -15. Change "$\operatorname{Hom}_R(P_n', C)$" to "$\operatorname{Hom}_R(P_{n-1}, C)$"

Page 853 diagram in middle of page, second row.

Change "$\prod \operatorname{Ext}^1(\sum P_k, B)$" to "$\prod \operatorname{Ext}^1(P_k, B)$"

Page 854 line -8 to -1. Should read:

Proposition 10.83.

(i) $\operatorname{Ext}_R^n(A, B)$ is a $Z(R)$-module. In particular, if R is a commutative ring, then $\operatorname{Ext}_R^n(A, B)$ is an R-module.

(ii) If A and B are left R-modules and $r \in Z(R)$ is a central element, then the induced map $\mu_r^*\colon \operatorname{Ext}_R^n(A, B) \to \operatorname{Ext}_R^n(A, B)$, where $\mu_r\colon B \to B$ is multiplication by r, is also multiplication by r. A similar statement is true in the other variable.

Proof. (i) By Example 10.47, μ_r is an R-map, and so it induces a homomorphism on $\operatorname{Ext}_R^n(A, B)$. It is straightforward to check that $x \mapsto \mu_r^*(x)$ defines a scalar multiplication $Z(R) \times \operatorname{Ext}_R^n(A, B) \to \operatorname{Ext}_R^n(A, B)$.

(ii) This follows from part (i), for we define scalar multiplication by r to be μ_r^*. •

Page 856 first line of proof of Corollary 10.86.

Change "Corollary 10.52" to "Corollary 10.75"

Page 857 lines 13, 14. Should read

is a well-defined function. Note that if ξ is a split extension, then $\psi([\xi]) = 0$. In order to prove that ψ is a bijection, we first analyze the diagram containing the map α.

Page 857 Statement of Lemma 10.87, first line.

Change "$X_1 \xrightarrow{i} X_0$" to "$X_1 \xrightarrow{j} X_0$"

Page 858 line 2. Should begin:

$$i: a \mapsto (a, 0) + S, \quad \beta: x_0 \mapsto (0, x_0) + S,$$

Page 858 line 16. Should begin (change i to i'):

square gives $\varepsilon x_0 = \eta' \beta' x_0 = -\eta' i' a = 0$.

Page 858 line 18. Should start:

$a = -\alpha x_1$. Replacing x_1 by $y_1 = -x_1$, we have

Page 860 lines 6 to 9. Should read

We begin by showing that θ is independent of the choice of cocycle α. If ζ is another representative of the coset $\alpha + \operatorname{im} d_1^*$, then there is a map $s: P_0 \to A$ with $\zeta = \alpha + s d_1$. But it is easy to see that the following diagram commutes:

Page 861 lines 3 to 5. Should read:

Proof. If every extension is split, then $|e(C, A)| = 1$, so that $|\operatorname{Ext}^1_R(C, A)| = 1$, by Theorem 10.89; hence, $\operatorname{Ext}^1_R(C, A) = \{0\}$. Conversely, if $\operatorname{Ext}^1_R(C, A) = \{0\}$, then Proposition 10.85 says that every extension is split. •

Page 864 line 5. Change "$\operatorname{Tor}^R_n(A, M)$" to "$\operatorname{Tor}^R_n(F, M)$"

Page 874 line −8. Change "$\ker\theta \subseteq G'$" to "$G' \subseteq \ker\theta$"

Page 877 line 10. Should read: $d_1: [x] \mapsto x[\ ] - [\ ]$;

Page 880 lines 7, 8. Should read:

d_n have the same formula as the maps in the bar resolution except that all symbols $[x_1|\cdots|x_n]$ now occur as $[x_1|\cdots|x_n]^*$; in particular, $[x_1|\cdots|x_n]^* = 0$ if some $x_i = 1$.

Page 881 line 8. Insert the following proof.

Proof. For $f \in \operatorname{Hom}_G(B_n, A)$, define $g: B_{n-1} \to A$ by

$$g(x_1|\cdots|x_{n-1}) = \sum_{y \in G} f(x_1|\cdots|x_{n-1}|y).$$

As in the proof of Theorem 10.21, sum the cocycle formula to obtain, for all $x_{n+1} = y$ in G,

$$\begin{aligned}(df)(x_1|\cdots|x_n|x_{n+1}) = x_1 f(x_2|\cdots|x_{n+1}) &+ \sum_{i=1}^{n-2} (-1)^i f(x_1|\cdots|x_i x_{i+1}|\cdots|x_{n+1}) \\ &+ (-1)^{n-1} f(x_1|\cdots|x_n x_{n+1}) + (-1)^n f(x_1|\cdots|x_n).\end{aligned}$$

In the next to last term, as x_{n+1} varies over G, so does $x_n x_{n+1}$. Therefore, if f is a cocycle, then $df = 0$ and

$$0 = x_1 g(x_2 | \cdots | x_{n-1}) + \sum_{i=1}^{n-2} (-i)^i g(x_1 | \cdots | x_i x_{i+1} | \cdots | x_n)$$
$$+ (-1)^{n-1} g(x_1 | \cdots | x_{n-1}) + m(-1)^n f(x_1 | \cdots | x_n)$$

(the last term is independent of x_{n+1}). Hence,

$$0 = dg + (-1)^n m f,$$

and mf is a coboundary. •

Page 883 line 10. Should read:

it suffices, by Proposition 7.49, to show that the

Page 883 line -15. Should be $(a, g) \in A \times G$

Page 883 line -4. Add Finally, d is a derivation because L is a homomorphism.

Page 884 Example 10.123(ii); add following

(ii) Suppose that G is an infinite cyclic group. Since every subgroup $S \subseteq G$ is cyclic, Theorem 10.122 gives $d(G) \leq 1$. If $d(G) = 0$, then $H^1(S, A) = \{0\}$ for all subgroups S and all modules A. In particular, if $S \cong \mathbb{Z}$ and $A \neq \{0\}$ is a trivial module, then $H^1(S, A) = \text{Der}(S, A)/\text{PDer}(S, A) \cong \text{Hom}(\mathbb{Z}, A) \neq \{0\}$. Hence, $d(G) = 1$.

Page 885 Add new corollary

Corollary 10.127. *A group* $G = \{1\}$ *if and only if* $\text{cd}(G) = 0$.

Proof. If $G = \{1\}$, then $\text{cd}(G) = 0$, by Example 10.123(i). Conversely, if $\text{cd}(G) = 0$, then $\text{cd}(S) = 0$ for every cyclic subgroup S of G. By Example 10.122(ii) and 10.122(iii), we have all $S = \{1\}$, and so $G = \{1\}$. •

Page 886 Delete present Corollary 10.127.

Page 889 lines 4, 5, 6 should be:

To have the associative law, we must have $u_\sigma(u_\tau u_\omega) = (u_\sigma u_\tau)u_\omega$; expanding this equation, the coefficient of each u_β is

$$\sum_\alpha g_\alpha^{\sigma,\tau} g_\beta^{\alpha,\omega} = \sum_\gamma g_\gamma^{\tau,\omega} g_\beta^{\sigma,\gamma}.$$

(The Einstein summation convention that suppresses Σ when one adds over a repeated index may clarify this equation.)

Pages 890, 891. Replace the paragraph beginning "We now show"

We now show that A is simple. Observe first that each u_σ is invertible, for its inverse is $f(\sigma^{-1},\sigma)^{-1}u_{\sigma^{-1}}$ (remember that im $f \subseteq E^\times$, so that its values are nonzero). Let I be a nonzero two-sided ideal in A, and choose a nonzero $y = \sum_\sigma c_\sigma u_\sigma \in I$ of shortest length; that is, y has the smallest number of nonzero coefficients. Multiplying by $(c_\sigma u_\sigma)^{-1}$ if necessary, we may assume that $y = u_1 + c_\tau u_\tau + \cdots$. Suppose that $c_\tau \neq 0$. Since $\tau \neq 1_E$, there is $a \in E$ with $a^\tau \neq a$. Now I contains $ay - ya = b_\tau u_\tau + \cdots$, where $b_\tau = c_\tau(a - a^\tau) \neq 0$. Hence, I contains $y - c_\tau b_\tau^{-1}(ay - ya)$, which is shorter than y (it involves u_1 but not u_τ). We conclude that y must have length 1; that is, $y = c_\sigma u_\sigma$. But y is invertible, so that $I = A$. Therefore, A is simple.

Page 893 lines 3–4. Should read

A ***discrete valuation*** on a field L is a function $\nu\colon L \to \Gamma \cup \{0\}$, where Γ is a multiplicative infinite cyclic group, such that, for all $a, b \in L$,

Page 893 After the definition of discrete valuation, say:

There is an equivalent definition of discrete valuation (in Exercise 11.15(ii) on page 921) in which Γ is an additive infinite cyclic group.

Page 893 line 19. Change "results where" to "results were"

Page 905 bottom diagram Change "$\widehat{\varphi}_0$" to "φ_0"

Page 907 line -3. Assume $0 \notin S$

Page 908 line 8. Assume $0 \notin S$

Page 911 line -9. Change "$\mu_S\colon S^{-1}m \to S^{-1}M$" to "$\mu_S\colon S^{-1}M \to S^{-1}M$"

Page 912 line -1. Change "*in M for some*" to "$= 0$ *in M for some* "

Page 913 line -10. Change "$f(a) \in \ker h_M$" to "$f(a) \in \ker h_B$"

Page 913 line -9. Change "$0 = \tau fa = f(\tau a)$" to "$0 = \tau f(a) = f(\tau a)$"

Page 919 Exercise 11.2. The exercise is false, Give a counterexample.

Page 921 Exercise 11.17(ii). Should read:

If $\mathcal{Z}(A)$ is the set of all zero divisors on an R-module A, prove that the complement of $\mathcal{Z}(A)$ is a saturated subset of R.

Page 926 line 17. Delete the last "not"

Page 928 line -12. Change "u^n" to "u^{n-1}"

Page 940 line -2. Change "$N(u) = a^2 - db^2$" to "$N(u) = a^2 - db^2 = u\overline{u}$, where $\overline{u} = a - b\sqrt{d}$."

Page 948 line 8. Change definition.

Definition. If $\alpha_1, \ldots, \alpha_n$ is an integral basis of $\mathcal{O}_E$, the ring of integers in an algebraic number field E, then the ***discriminant*** is

$$\Delta(\mathcal{O}_E) = \det[\operatorname{tr}(\alpha_i \alpha_j)]$$

(one can prove that this definition does not depend on the choice of integral basis).

Page 950 line -10. Should end: then it is easy to

Page 950 line -6. Should end: If I is a nonzero fractional ideal, then

Page 953 After the statement of Theorem 11.95, remark that the converse is also true.

Page 958 Exercise 11.46. Change "$a \in J$" to "$a \in I$"

Page 958 Exercise 11.53(i). Change "Prove that $\mathbb{Z} \times \mathbb{Z}$ is not a principal ideal ring." to "Prove that $\mathbb{Z} \times \mathbb{Z}$ is a principal ideal ring."

Page 960 line 1. Add hypothesis: *Let R be a domain*.

Page 963 line -2. Change "$\{m \in M :$" to "$\{m \in T :$"

Page 967 line 1. Change "$I_1 \oplus \cdots I_n$" to "$I_1 \oplus \cdots \oplus I_n$"

Page 967 line 2. Change "$I_1' \oplus \cdots I'n$" to "$I_1' \oplus \cdots \oplus I_n'$"

Page 978 line -3. Add: From this point on, we will consider only commutative rings.

Page 993 line -12. Add a footnote, saying that there is actually equality $\operatorname{ht}(\mathfrak{p}/(x)) + 1 = \operatorname{ht}(\mathfrak{p})$.

Page A-8. Delete the bottom three lines of text.