Solutions

1. You roll a standard die seven times and count the number of times four appears.

   (a) (1 point) Is this experiment a Bernoulli process?

   **Solution:** It is a Bernoulli process, because

   - it consists of seven independent trials,
   - each trial has two outcomes (four or not four),
   - and each trial has the same probability of success, \( \frac{1}{6} \).

   (b) (3 points) What is the probability of that exactly five fours were rolled?

   **Solution:**

   \[
   \Pr(\text{Exactly five fours}) = C(7, 5) \left( \frac{1}{6} \right)^5 \left( \frac{5}{6} \right)^2 = \frac{7!}{5! \cdot 2!} \cdot \frac{5^2}{6^7} = 21 \cdot \frac{5^2}{67}
   \]

   (c) (3 points) What is the probability that at most two fours were rolled?

   **Solution:**

   \[
   \Pr(\text{At most two fours}) = \Pr(\text{Exactly zero fours}) + \Pr(\text{Exactly one four}) + \Pr(\text{Exactly two fours})
   \]

   \[
   = C(7, 0) \left( \frac{1}{6} \right)^0 \left( \frac{5}{6} \right)^7 + C(7, 1) \left( \frac{1}{6} \right)^1 \left( \frac{5}{6} \right)^6 + C(7, 2) \left( \frac{1}{6} \right)^2 \left( \frac{5}{6} \right)^5
   \]
2. An experiment consists of spinning the arm on the following spinner.

You are offered a bet on the outcome of this experiment. You win $1 if the arm finishes in the black region, lose $1 if the arm finishes in the gray region, and win $0 if the arm finished in the white region. The random variable $X$ is defined to be the amount you win as a result of the bet.

(a) (4 points) Complete the following table showing the distribution of $X$.

<table>
<thead>
<tr>
<th>Outcome $X$</th>
<th>White</th>
<th>Black</th>
<th>Gray</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Pr($X = x$)</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

(b) (3 points) Compute the expected value of $X$.

Solution:

$$E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} + (-1) \cdot \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}.$$ 

(c) (1 point) Should you take this bet?

Solution: Since the expected value is negative, you should not take the bet.