6.4.2 \( \frac{1}{2} \) \: \text{Urn I} \quad \frac{1}{3} \: \text{W} \quad \frac{3}{8} \: \text{R} \quad \frac{5}{8} \: \text{W} \quad (a) \: P_r(\text{Urn I} \mid R) = \frac{\frac{1}{2} \cdot \frac{3}{8}}{\frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{16}{23} \\
\frac{1}{2} \: \text{Urn II} \quad \frac{3}{8} \: \text{R} \quad \frac{5}{8} \: \text{W} \quad (b) \: P_r(\text{Urn II} \mid R) = \frac{\frac{1}{2} \cdot \frac{3}{8}}{\frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{1}{3}} = \frac{7}{23} \\
6.4.6 \quad (a) \: P_r(I \mid B) = \frac{(0.6)(0.8)}{(0.6)(0.8) + (0.3)(0.3) + (0.1)(0.4)} = \frac{48}{61} \\
\quad (b) \: P_r(III \mid B) = \frac{(0.3)(0.3)}{(0.6)(0.8) + (0.3)(0.3) + (0.1)(0.4)} = \frac{9}{61} \\
\quad (c) \: P_r(III \mid B) = \frac{(0.1)(0.4)}{(0.6)(0.8) + (0.3)(0.3) + (0.1)(0.4)} = \frac{4}{61} \\
6.4.14 \quad \text{Let} \quad R_1, R_2 \quad \text{be the events of choosing a red ball from Urn I and Urn II, respectively.} \\
\quad W_1, W_2 \quad \text{be the events of choosing a white ball from Urn I and Urn II, respectively.} \\
\begin{align*}
\frac{5}{7} \: R_1 \quad \frac{5}{8} \: R_2 \\
\frac{2}{7} \: W_1 \quad \frac{4}{8} \: R_2 \quad \frac{4}{8} \: W_2 \\
\end{align*} \\
\quad P(W_1 \mid W_2) = \frac{\frac{2}{7} \cdot \frac{4}{8}}{\frac{5}{7} \cdot \frac{3}{8} + \frac{2}{7} \cdot \frac{4}{8}} = \frac{8}{23}
5.2.4
(a) $4 \cdot 2 = 8$
(b) 

\[ \begin{array}{c}
B \\
R \\
C \\
R \\
P \\
R \\
D \\
\end{array} \]

5.2.8
(a) $4 \cdot 2 \cdot 2$
(b) 

\[ \begin{array}{c}
M_1 \\
H_1 \\
S_1 \\
S_2 \\
H_2 \\
S_1 \\
S_2 \\
M_2 \\
H_1 \\
S_1 \\
S_2 \\
H_2 \\
S_1 \\
S_2 \\
M_3 \\
H_1 \\
S_1 \\
S_2 \\
H_2 \\
S_1 \\
S_2 \\
M_4 \\
H_1 \\
S_1 \\
S_2 \\
H_2 \\
S_1 \\
S_2 \\
\end{array} \]
5.2.14
(a) $5 \times 32 \times 6 \times 2$
(b)
5.2.16.
(a) with replacement
\[5 \cdot 5 \cdot 5 \cdot 5 = 5^4\]
(b) without replacement
\[5 \cdot 4 \cdot 3 \cdot 2 = P(5, 4)\]

5.2.30. \[6! \cdot 4!\]

5.3.8 \[C(7, 3) = \frac{7!}{4! \cdot 3!} = 35\]

5.3.16.
\[C(8, 5) = 56\]
\[C(8, 3) = 56\]

5.3.20. \[C(9, 3) \cdot C(8, 6) = 2352\]

5.3.26. We first distribute the I's, there \[C(8, 3)\]
ways to choose 3 of 8 positions. Then we arrange
the letters V, R, G, N, A in the remaining 5 positions,
this can be done in \[P(5, 5) = 5!\] ways.
Thus, the number of different words is
\[C(8, 3) \cdot P(5, 5) = \binom{8}{3} \cdot 5! = 6720,\]
6.1.36. The sample space \( S \) is the set of all ordered 5-tuples of numbers 1 to 6. \( n(S) = 6^5 \).
Consider the event \( E \) of when exactly 2 fours will occur. There are \( \binom{5}{2} \) ways to choose 2 of 5 positions in which to put the fours. Then, there cannot be four in the other 3 positions, this can be done in \( 5^3 \) ways.
Thus, \( n(E) = \binom{5}{2} \cdot 5^3 \).
So, \( P(E) = \frac{n(E)}{n(S)} = \frac{\binom{5}{2} \cdot 5^3}{6^5} \approx 0.16 \).

6.1.40. The number of possible poker hands is
\[ n(S) = \binom{52}{5} = 2,598,960. \]
There are 13 ways to choose a rank for the 4 cards with the same rank. Then there are \( 52-4=48 \) cards left to choose another card. Thus,
\[ n(E) = 13 \cdot 48. \]
So, \( P(E) = \frac{n(E)}{n(S)} = \frac{13 \cdot 48}{2,598,960} \approx 0.00024. \)

6.2.10 \( \Pr( \text{at least 2 sixes}) = 1 - \Pr( \text{less than 2 sixes}) \)
\[ \Pr( \text{no six}) = \frac{5^4}{6^4} \]
\[ \Pr( \text{exactly 1 six}) = \binom{4}{1} \frac{5^3}{6^4} = \frac{4}{6^4} \]
Then \( \Pr( \text{less than 2 sixes}) = \frac{5^4}{6^4} + \frac{4}{6^4} = \frac{9}{6^4} \approx 0.87. \)
So, \( \Pr( \text{at least 2 sixes}) \approx 1 - 0.87 = 0.13. \)
(a) \( P_r (\text{the first is } \#2 \text{ and the second is } \#2) \)
\[
= \frac{16 \cdot 15}{24 \cdot 23}
\]

(b) \( P_r (\text{the first is } \#1 \text{ and the second is } \#1) \)
\[
= \frac{8 \cdot 7}{24 \cdot 23}
\]