Week 7: 1.3–1.4

Linear Models

In this activity, you will model various situations using linear equations; in other words, equations like \( ax + by = c \), or \( y = mx + b \).

1. A company manufactures sofas. It has a fixed setup cost of $4,500, and each sofa costs $600 to make. Each sofa sells for $750.

a) How much would it cost to manufacture 20 sofas?

\[ C(20) = 4500 + 600(20) = 4500 + 12000 = 16500 \text{ dollars} \]

b) How much would it cost to manufacture \( x \) sofas?

\[ C(x) = 4500 + 600x \text{ dollars} \]

c) How much revenue is generated by selling 20 sofas?

\[ R(20) = 750(20) = 15,000 \text{ dollars} \]

d) How much revenue is generated by selling \( x \) sofas?

\[ R(x) = 750x \text{ dollars} \]

e) How much would the company make or lose by selling 20 sofas? What about 3,000?

\[ P(20) = R(20) - C(20) = 15,000 - 16,500 = -1,500 \]

The company would lose $1500 by selling 20 sofas.

f) How much would the company make or lose by selling \( x \) sofas?

\[ P(x) = R(x) - C(x) = 750x - (4500 + 600x) = 150x - 4500 \text{ dollars} \]

2. Suppose that a company that manufactures an item has a fixed setup cost of \( F \) dollars, a cost of \( c \) dollars per item manufactured, and sells each item for \( p \) dollars. The manufacturer makes \( x \) units.

a) The cost \( C(x) = F + CX \).

b) The revenue \( R(x) = PX \).

c) The profit \( P(x) = (p-c)x - F \).

\[ P(3,000) = R(3,000) - C(3,000) = 150(3,000) - 4500 = 450000 - 4500 = 445,500 \]

The company would make $445,500 by selling 3,000 sofas.
3. Using problem 1, answer the following.

a) What is the profit if 5,000 sofas are sold?

\[ P(5,000) = 150(5,000) - 4500 = 75,000 - 4500 \]
\[ = \boxed{74,500 \text{ dollars}} \]

b) How many sofas must be sold to have a profit of $12,000?

\[ 12,000 = 150x - 4500 \]
\[ 16,500 = 150x \]
\[ x = \frac{16,500}{150} = 110 \text{ sofas} \]

4. The graphs of \( C(x) \), \( R(x) \), and \( P(x) \) are show below. Label each line according to which function it graphs.

5. The graphs of which of these functions must always pass through the origin? Why?

\[ R(x) \]
If you sell 0 units of the product, your revenue will always be 0 dollars.
6. Suppose that there is a demand for 7,500 computer monitors when their selling price is $800 per monitor, and that there is demand for 9,000 when the price drops to $700. Assume that the relationship between price and demand is linear. Let \( x \) represent the demand in units, and \( y \) the price in dollars. (We assume that \( x \) is the independent variable.)

a) What is the value of \( x \) when \( y \) is 750?

\[ x = 7500 \]

b) What is the value of \( y \) when \( x \) is 9,000?

\[ y = 700 \]

c) Find the slope of the line determined by these two points.

\[
(7500, 750) \quad m = \frac{750 - 700}{7500 - 9000} = \frac{50}{-1500} = \frac{-5}{150} = \frac{-1}{30}
\]

**Formula:** Recall that the equation for a line passing through a point \((x_1, y_1)\) with slope \( m \) is \( y - y_1 = m(x - x_1) \). This is called the **point-slope** form of a line.

\[ y - 750 = \frac{-1}{30}(x - 7500) \]

**Definition:** A demand equation expresses the relationship between the selling price of an item and the quantity demanded by consumers at that price.

7. Should the values of \( x \) in a demand equation ever be negative? What about the values of \( y \)?

The values of \( x \) should not be negative because consumers cannot demand a negative quantity. The values of \( y \) should never be negative because the selling price of an item would not be negative.

8. Sketch the graph of the function from 6(d). Must all demand equations slope in the same direction?

\[
y - 750 = \frac{-1}{30}x + \frac{7500}{30} \quad (0, 10)
\]

\[
y - 750 = \frac{-1}{30}x + 250
\]

\[
y = \frac{-1}{30}x + 1000 \quad (0, 1000)
\]

Yes, all demand equations will have negative slope.
9. Suppose that the manufacturer will not market its monitors if the price is $600 or less. For every $50 above $600, the manufacturer will market 1,000 monitors. Assume that all monitors that are marketed are sold, and that the relationship between price and the number marketed is linear. Let $x$ represent the supply in units, and $y$ the price in dollars.

a) What is the value of $x$ when $y$ is $600$? What about $650$?

When $y = 600$, $x = 0$.

When $y = 650$, $x = 1000$.

b) Find the equation of the line passing through the two points you just found. This is the supply equation.

\[
\begin{align*}
\text{Definition: A supply equation expresses the relationship between the selling price of an item and the quantity supplied by manufacturers.}
\end{align*}
\]

10. Should the values of $x$ in a supply equation ever be negative? What about the values of $y$?

The values of $x$ should never be negative because the supplier cannot supply a negative quantity of units.

The values of $y$ should never be negative because the price of an item would not be negative.

11. Sketch the graph of the function found above. Must all supply equations slope in the same direction?

Yes, all supply equations will have positive slope.
12. Suppose that there is a demand for 5,600 picture frames when they sell for $33, and 6,900 will be supplied. When the unit price decreases by $12, there is a demand for 1,600 more frames, and manufacturers will supply 3,600 fewer frames. Find and sketch the supply and demand equations.

Demand equation:
\[ y - 33 = \frac{3}{400} (x - 5600) \]

Supply equation:
\[ y - 33 = \frac{1}{300} (x - 6900) \]
Creating Linear Systems

Definition: A linear system is a collection of linear equations.

1. Suppose that an apartment complex is being developed that will have small, large, and luxury apartments. There will be a total of 192 apartments. The number of small apartments will equal the number of large and luxury together. The number of small apartments will be three times the number of luxury apartments. Express this situation mathematically in the following steps:

a) Determine what the variables represent. (You will need to define three variables.)

Let \( s \) denote the number of small apartments, \( l \) denote the number of large apartments, and \( L \) denote the number of luxury apartments.

b) Using these variables, express the relationships among the types of apartments. (You should have three equations.)

\[
\begin{align*}
\text{1. } & s + l + L = 192 \\
\text{2. } & s = l + L \\
\text{3. } & s = 3L
\end{align*}
\]
2. Suppose that you have $86 in one-, five-, and ten-dollar bills. You have a total of 27 bills, and twice as many fives as ones. Express this situation mathematically in the following steps:

a) Determine what the variables represent. (You will need to define three variables.)

Let \( o \) denote the number of one-dollar bills,

\[ o + f + t = 27 \]

f denote the number of five-dollar bills,

\[ 1 \cdot o + 5 \cdot f + 10 \cdot t = 86 \]

and \( t \) denote the number of ten-dollar bills.

b) Using these variables, express the relationships among the types of bills.
3. Suppose you have $8900 to invest in three companies. Shares in company A cost $50 each and pay $1 each per year in dividends. Shares in company B cost $90 and pay $2 per year. Shares in company C cost $40 and pay $1.60. You want to invest half as much money in company C as in B, and you want to earn $222 per year. Express this situation mathematically in the following steps:

a) Determine what the variables represent.

Let a denote the number of shares in company A,

b denote the number of shares in company B,

c denote the number of shares in company C.

b) Using these variables, express the relationships among the shares in different companies.

\[ 50a + 90b + 40c = 8900 \]
\[ 1.60c = \frac{1}{2} 90b \]
\[ 1 \cdot a + 2b + 1.60c = 222 \]