Math 124 Exam III

Exam Information

• This exam has 10 questions for a total of 100 points.
• Partial credit will be given for partially correct work.
• Circle your answers.
• Anyone caught cheating will receive an automatic zero for this exam, and there may be more severe consequences.
• No calculators, phones, or other electronic devices may be used during the exam.
• You have 60 minutes to complete the exam.

I certify that I have read, understand, and agree to abide by the above rules.

Signature: ____________________________

Circle your section number.

<table>
<thead>
<tr>
<th>Section</th>
<th>Instructor</th>
<th>Meeting Time</th>
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<tr>
<td>M1</td>
<td>Austin Rochford</td>
<td>Tuesdays 9:30–10:50 A.M.</td>
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<tr>
<td>M2</td>
<td>Katie Anders</td>
<td>Thursdays 9:30–10:50 A.M.</td>
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<tr>
<td>Q1</td>
<td>Austin Rochford</td>
<td>Tuesdays 12:30–1:50 P.M.</td>
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<tr>
<td>Q2</td>
<td>Alison Reddy</td>
<td>Thursdays 12:30–1:50 P.M.</td>
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<tr>
<td>S1</td>
<td>Panupong Vichitkunakorn</td>
<td>Tuesdays 2:00–3:20 P.M.</td>
</tr>
<tr>
<td>S2</td>
<td>Panupong Vichitkunakorn</td>
<td>Thursdays 2:00–3:20 P.M.</td>
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DO NOT TURN THE PAGE UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

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<tr>
<th>Question</th>
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1. (3 points) Find the augmented matrix of the following system of equations \textbf{without actually solving the problem}.

\begin{align*}
5x + 6y + 7z &= 1 \\
4x - 3y &= -2 \\
7x - 2y + 50z &= 27
\end{align*}

2. (6 points) For each of the following matrices, determine whether the corresponding linear system is consistent or inconsistent. \textbf{Explain briefly without actually solving the problem}.

(a) \[
\begin{bmatrix}
1 & 0 & -3 & 2 \\
0 & 1 & 2 & 4 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
1 & 0 & -3 & 2 \\
0 & 1 & 2 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
3. (8 points) Give the solution to the system of equations that has the following augmented matrix. You may use any method you choose. Use variable names $x_1, x_2, x_3$.

$$
\begin{bmatrix}
1 & -3 & 5 & -5 \\
0 & 1 & 4 & -2 \\
0 & 0 & 1 & 2
\end{bmatrix}
$$
4. (8 points) For what value of $k$ is the linear system corresponding to the following augmented matrix consistent?

\[
\begin{bmatrix}
1 & -6 & 2 & | & 8 \\
4 & -25 & 10 & | & 6 \\
2 & -13 & 6 & | & k
\end{bmatrix}
\]
5. (14 points) The system of equations

\[ \begin{align*}
  x - 2y + 2z &= 3 \\
  -3x + 6y - 5z &= -4
\end{align*} \]

has the augmented matrix

\[
\begin{bmatrix}
  1 & -2 & 2 & | & 3 \\
  -3 & 6 & -5 & | & -4
\end{bmatrix}
\]

(a) Give the reduced row echelon form of the matrix. Clearly label what you are doing in each step.

(b) Give the parametric solution to the system of equations.

(c) Give a particular solution to the system of equations.
6. (14 points) Find

(a) \[
\begin{bmatrix}
2 & -1 \\
-2 & 5 \\
1 & 0
\end{bmatrix}
+ 2 \begin{bmatrix}
-1 & 1 \\
2 & -2 \\
0 & 3
\end{bmatrix}.
\]

(b) \[
\begin{bmatrix}
1 & -1 \\
-2 & 3 \\
5 & -3
\end{bmatrix}^\top
\]

(c) \[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix}
\times \begin{bmatrix}
-1 \\
3 \\
-2
\end{bmatrix}
\]
7. (12 points) Consider the following matrices with their orders shown.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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</thead>
<tbody>
<tr>
<td>$3 \times 2$</td>
<td>$2 \times 2$</td>
<td>$2 \times 4$</td>
<td>$4 \times 3$</td>
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</tbody>
</table>

Determine whether each of the following exists or not. For those that do, determine the order. For those that do not, state why not.

(a) $AB$

(b) $AD$

(c) $ACD$

(d) $D^{-1}$

8. (8 points) Let $A$ and $B$ be the coefficient matrix and the equation constant vector for the matrix equation $AX = B$. Find the solution vector $X = A^{-1}B$.

$$A = \begin{bmatrix} 3 & 4 \\ 8 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
9. (18 points) A remote island economy has 2 industries: papayas and mangos. Producing $1 of papayas requires $0.40 of papayas and $0.10 of mangos. Producing $1 of mangos requires $0.20 of papayas and $0.80 of mangos. A nearby island demands $3000 of papayas and $1000 of mangos.

(a) Determine the consumption matrix, $A$.

(b) Determine the demand matrix, $D$.

(c) Compute $I - A$. 

(d) Find the inverse \((I - A)^{-1}\). Simplify your answer.

(e) Determine the production vector for the given external demand.

(f) Determine if the economy is productive.
10. (9 points) Given matrix $A$, find the inverse matrix $A^{-1}$, if it exits. Clearly label what you are doing in each step.

$$A = \begin{bmatrix} 1 & 2 & -3 \\ -1 & -1 & 4 \\ 1 & 4 & 0 \end{bmatrix}$$