Math 285 D1

Homework 1

(Homework is due on 6/21/2016)

1. Solve, i.e., find the general solution of, the differential equation

\[ y' = \frac{x^2 + x^2y^2}{y + xy}. \]

[8 pts]

2. By using a suitable substitution find the solution of the differential equation \( y' = e^{x+y} - 1 \) which satisfies the side condition \( y(0) = 0 \).

[8 pts]

3. Solve the differential equation \( (x^2 - y^2)y' = xy \).

[10 pts]

4. Solve the differential equation

\[ 3xy^2y' = 3x^3 \sin x + y^3. \]

[10 pts]

5. A tank is generated by rotating the curve \( y = x^2 \) about the \( y \)-axis. The tank contains water with depth \( d \) ft. A small plug at the bottom of the tank with cross sectional area \( a \) is removed and the tank is allowed to drain. [See class notes].

(a) Find a formula for the depth of water in the tank at time \( t \), and hence the time the tank takes to drain, in
terms of $d, a$ and $g = 32$ ft per sec$^2$ (the acceleration due to gravity).

(b) It is found that the tanks drains in 81 seconds when the initial depth was 4 ft. What is the area of the plug? [14 pts]