For Section 7.3 we will practice the odd numbered exercises from the textbook. Below is a selection of problems, but I encourage you to practice the rest at home.

1. In a class 16 people live on campus and 8 live off-campus. If we were to draw one student’s name from a hat, what is the probability that the student is living off-campus?

\[
\text{Total number of students: } 16 + 8 + 24. \text{ The probability of off-living on campus is then,} \\
\frac{\text{# of students living off-campus}}{\text{Total number of students}} = \frac{8}{24} = \frac{1}{3}
\]

11. Determine the probability of the prince ending up in Room A and Room B in the given scenario.

We can figure out the probability of ending up in each room by considering the probabilities of taking each path.

There are two paths to Room B, the probability of taking each is given by \(\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}\).

The probability of ending up in Room B is \(\frac{1}{6} + \frac{1}{6} = \frac{1}{3}\).

We can find the probability of ending up in Room A by computing its complement,

\[1 - P(\text{ending up in Room A}) = 1 - \frac{1}{3} = \frac{2}{3}\]

17. Consider the following game for two people: A player rolls two standard dice and makes a proper fraction with the two numbers.

- If the fraction is in simplest form, the person who rolled wins.
- If the fraction is not in simplest form the other person wins.
- if the two numbers are identical the other person wins.

Is this a fair game? Why or why not?

We find all possible outcomes rolling two dice are given by,

\[(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\]

We notice that there are 22 ways we can obtain a fraction in its simplest form, 6 ways to obtain identical numbers and 8 ways where we obtain a fraction not in simplest from.

So one player has a probability of winning of \(\frac{22}{36}\), while the other player only has a probability of winning of \(\frac{14}{36}\). This is not a fair game.
21. If we spin the spinners below, what is the probability of getting different numbers?

We see that there are 9 possible outcomes,
$$\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$
out of which only 6 have different numbers, so the probability
is \( \frac{6}{9} = \frac{2}{3} \).

25. In each of game below, explain whether or not it is a fair game. If it is not a fair game modify it so that it becomes fair.

(a) Two players take turns rolling two dice and adding the numbers. Player A wins if the number is even, player B wins if the number is odd.

We see that the chances of getting an odd and an even sum are each \( \frac{18}{36} \). So it is a fair game.

(b) Two players take turns rolling two dice and multiplying the numbers. Player A wins if the number is even, player B wins if the number is odd.

One way to think about it is to notice that,
\[
\begin{align*}
\text{even} \times \text{even} &= \text{even}, \\
\text{even} \times \text{odd} &= \text{even}, \\
\text{odd} \times \text{even} &= \text{odd}, \\
\text{odd} \times \text{odd} &= \text{even}.
\end{align*}
\]
This tells us that the chances of Player A’s winning are higher than Player B’s. We can make the game fair by giving 1 point to Player A when they win and 3 points to Player B when they win. (Why?)

(c) Two players take turns rolling three dice and adding the numbers. Player A wins if the number is even, Player B wins if the number is odd.

33. If you flip a coin 10 times, what is the probability it is at least 4 heads?

The probability when you flip a coin ten times and getting 4 heads is the same as the probability of having 4, 5, 6, 7, 9, and 10 heads after the 10 flips respectively. We can rephrase this question (to save us some time) as what is the probability of having at most three heads using the fact that,
\[
P(\text{At least 4 heads}) = 1 - P(\text{At most 3 heads})
\]
\[
= 1 - (P(1 \text{ head}) + P(2 \text{ heads}) + P(3 \text{ heads}))
\]
To determine each probability we need to now the number of ways we could get each favorable outcome:
1 head: 10 ways, 2 heads: \( \frac{10 \times 9}{2 \times 1} = 45 \), 3 heads: \( \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120 \)
The number of possible outcomes will be \( 2^{10} \) (why?). So our final answer is,
\[
P(\text{At least 4 heads}) = 1 - \frac{10 + 45 + 120}{1024} = 1 - \frac{175}{1024} = \frac{849}{1024} \approx 0.83
\]
37. Consider a pair of triangular pyramid-shaped dice (has four sides with numbers from 1 through 4). If you roll two dices at a time, what is the most likely sum and what is the probability of that sum?

<table>
<thead>
<tr>
<th>Rolls:</th>
<th>2, 3, 4, 5</th>
<th>The most likely sum is a 5.</th>
</tr>
</thead>
</table>
| (1,1) , (1,2), (1,3), (1,4) | 3, 4, 5, 6 | The probability of obtaining a 5 will be \( \frac{4}{16} = \frac{1}{4} \).
| (2,1), (2,2), (2,3), (2,4) | 4, 5, 6, 7 |
| (3,1), (3,2), (3,3), (3,4) | 5, 6, 7, 8 |
| (4,1), (4,2), (4,3), (4,4) | Possible sums: |

39. In the board game Monopoly, if you roll three doubles in a row you go to jail.

(a) What is the probability of rolling three doubles in a row?

We know that the probability of rolling a double is \( \frac{6}{36} = \frac{1}{6} \). So, the probability of rolling doubles three times in a row is \( \left( \frac{1}{6} \right)^3 = \frac{1}{216} \).

(b) Describe three ways in which you could represent this probability.

We could represent this probability as a fraction \( \frac{1}{216} \), as a decimal 0.00462, or you could say there is 1 out of 216 chance of rolling three doubles.

(c) Say you are writing a news article and want to pick the most understandable representation for the general population. Which one would you choose and why?

Reflection: What were some of the big ideas or strategies you used for each problem?