1. Out of 40 students, 14 are taking English, 29 are taking Chemistry, and 5 are taking both. Use a Venn Diagram to answer the following questions:

(a) How many students are taking only English? \[14 - 5 = 9\]
(b) How many students are taking only Chemistry? \[29 - 5 = 24\]
(c) How many students are taking neither class? \[40 - (9+5+24) = 40 - 38 = 2\]

2. Let \(A = \{x \mid x \text{ is female}\}\), \(B = \{x \mid x \text{ has brown hair}\}\), \(C = \{x \mid x \text{ has blue eyes}\}\). Represent the following descriptions with a Venn diagram and also with symbols:

(a) A female with blonde hair and blue eyes. \[B \cap (A \cap C)\]

(b) A male with brown hair and brown eyes. \[(A \cup C) \cap B\]
3. How many subsets of \{a, b, c\} are there?

The set has three elements so the number of subsets is given by \(2^3 = 8\).

4. What is the greatest 4-digit number in base 8?

In base 8, the largest number is 7777.

5. Convert the following numbers to binary:

(a) \(32_{10}\)

\[
32 = 2^5 = 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 100000_2.
\]

(b) \(56_7\)

\[
56_7 = 5 \times 7^1 + 6 \times 7^0 = 41_{10} = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 101001_2.
\]

(c) \(3e2_{16}\)

\[
3e2_{16} = 3 \times 16^3 + 14 \times 16^2 + 2 \times 16 + 11 \times 16^0,
\]

Using the fact that \(2^4 = 16\) we can rewrite all the numbers in terms of powers of 2. Expanding everything out and ordering the terms from higher power to lowest we get,

\[
= (2 + 1) \times 2^{12} + (2^3 + 2^2 + 2)2^8 + 2 \times 2^4 + (2^3 + 2 + 1)
= 2^{13} + 2^{12} + 2^{11} + 2^{10} + 2^9 + 2^5 + 2^3 + 2 + 2^0
\]

Any power of two that is missing we can think of as being multiplied by 0,

\[
= 1 \times 2^{13} + 1 \times 2^{12} + 1 \times 2^{11} + 1 \times 2^{10} + 1 \times 2^9 + 0 \times 2^8 + 0 \times 2^7 + 0 \times 2^6
+ 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0
= 11111000101011_2
\]

**Question:** Can you think of a way to convert (b) to binary without converting to base 10 first?
6. In what base do the following equations hold?

(a) \(10_x + 10_x = 100_x, \ [x=2]\)

(b) \(8_x + 8_x = 10_x, \ [x=16]\).

**Question:** What strategies did you use to find the base in the previous problem?