Math 231 – Calculus II
Exam II
Friday, March 21, 2014

Scoring Information:

• This exam has 6 questions for a total of 100 points. The point values are listed in parenthesis at the beginning of the problem.

• You must show all work in order to receive full credit for a problem. Points may be deducted at the discretion of the instructor for an answer not supported by a reasonable amount of work.

• Anyone caught cheating will receive an automatic zero for this exam. This includes looking at other students’ exams.

• Anyone who does not stop working immediately when time is up will receive an automatic zero on the exam.

Further Exam Information:

• Please neatly work in the space provided for each problem. Should you feel the need to use more than the provided space, you may work on the back of the page.

• You may not use a calculator on this exam.

• Please turn off and put away your cell phone.

• If you finish with more than 5 minutes remaining, you may turn in your exam and leave. If you finish within the last 5 minutes, please turn in your exam, but remain seated until time is up.

DO NOT TURN THE PAGE UNTIL TOLD TO DO SO BY YOUR INSTRUCTOR.

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1. For this problem, consider the sequence \( \{a_n\} \) for which \( a_n = 10 - 2 \left( \frac{1}{n} \right) \) for \( n \geq 1 \).

(a) (5 points) Write out the first five terms to this sequence (starting with \( n = 1 \)).

(b) (2 points) Is this sequence bounded? Justify your answer.

(c) (2 points) Is this sequence monotonic? Justify your answer.

(d) (3 points) Does this sequence converge? If it does, identify the value to which it converges. Justify your answer.

(e) (2 points) Does the series given by \( \sum_{n=1}^{\infty} a_n \) converge or diverge? Justify your answer.
2. (a) (2 points) If \( \lim_{n \to \infty} a_n = \frac{1}{4} \), what can we say about \( \sum_{n=1}^{\infty} a_n \)? Briefly explain.

(b) (2 points) If \( \sum_{n=0}^{\infty} a_n = 2 \), what can we say about \( \lim_{n \to \infty} a_n \)? Briefly explain.

(c) (2 points) If \( \sum_{n=1}^{\infty} \frac{1}{a_n} \) converges, then what can we say about \( \sum_{n=1}^{\infty} a_n \)? Briefly explain.

(d) (2 points) If \( \sum_{n=1}^{\infty} a_n \) converges, will \( \sum_{n=1}^{\infty} |a_n| \) converge? Briefly explain.
3. (14 points) Use the Integral Test to determine whether the series

\[ \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \]

converges or diverges. You must verify the conditions of the Integral Test before using it. Use proper terminology and show all work.
4. Determine whether each of the following series converges or diverges. If a series converges, determine the value to which it sums. If a series diverges, state why. Use proper terminology and show all work.

(a) (10 points) \[ 2 - \frac{6}{4} + \frac{18}{16} - \frac{54}{64} + \cdots \]

(b) (10 points) \[ \sum_{n=16}^{\infty} \frac{1}{\sqrt{n}} \]
Problem continued from previous page.

(c) (10 points) \[
\sum_{n=2}^{\infty} \left( \frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n-1}} \right)
\]

5. (10 points) Determine whether the following series converges or diverges. Use proper terminology and show all work.

\[
\sum_{n=1}^{\infty} \frac{e^{-n}}{n^2}
\]
6. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Use proper terminology and show all work.

(a) (12 points) \[ \sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n+7} \]

(b) (12 points) \[ \sum_{n=1}^{\infty} \frac{(-3)^n+1ne^n}{n!} \]