Mathematics is the poetry of logic, the source of endlessly beautiful and intricate patterns, intertwined with almost conspiratorial reverberations. Mathematics is also the language in which different scholarly disciplines produced their quantitative models of the world; even qualitative models depend on what “counts”.

In the third or fourth set of homework papers I ever graded, one student successfully solved a tricky problem and wrote, over and over again at the end of the page: “Math is groovy”. (This was a while ago.) Math is still groovy to me. And what could be a more rewarding way to spend your time than talking about something you love to smart young people who want and or need to learn it? The University is an institution which is intellectually consecrated to the creation, study and preservation of facts, knowledge and wisdom. I really mean that! At the same time, our undergraduate program does not exist for the exaltation of the professor, but for the education of the student. As the years go by, the greatest challenge in teaching becomes finding the creative empathy to imagine afresh what a subject looks like to a student, when you yourself have known it well for 35 or 40 years. There is no such thing as a successful course in isolation; its content must be matched with its participants. When you are giving a lecture and suddenly the world disappears except for you and your theorems, harmoniously thrumming with your voice and your writing on the blackboard, you need to know that you’ve lost your students and are providing them, at best, with amusement. A more productive self-satisfaction occurs when you can successfully answer a volley of questions from different students at different levels.

Teaching involves both large-scale and small-scale planning, and even in as established a field as mathematics, the prevailing outside circumstances can change. Consider the nature and importance of computation. The curriculum was largely designed to domesticate deep mathematical results into algorithms which could be carried out by hand on paper. This is no longer a particularly useful skill. The invention of the typewriter eliminated the need for most people to have clearly comprehensible handwriting; the invention of the calculator did the same for arithmetic, and the proliferation of computer algebra systems is doing the same for the algorithms of undergraduate mathematics. Complicated hand-computations should rapidly become a thing of the past. This will cause much undergraduate glee until they realize that this means a proliferation of word problems.

The person who can open the most locks is not the one with the largest key-ring, but the one with the most powerful master-key. Most arguments about curriculum founder on the belief that if a particular idea is not developed in a particular way, then the students will miss something essential in their education. But if we assume that students will learn a subject if (and only if) it is explicitly in the syllabus, then we have conceded that we are so inadequately educating our students that they won’t be able to pick things up on their own if they need it. In my seminar course Math 496, I introduce my students to mathematical
research, not by a set of rules for problem-solving or by running them through a canned list of topics with a predestined outcome, but by teaching them how to think about asking questions about the mathematics they already know and letting them realize how close they already are to the frontiers of knowledge.

At the first class meeting, I always begin with a rigorously true sentence: “It is a privilege to teach at the University of Illinois and it is a pleasure to teach this course.” Most of the other details of how I run my classes can be found in my teaching guide “Chalking It Up”, and, since most of my students will become professionals of some kind (and therefore will do some teaching, at least informally), I’ll give them the link to this guide and invite them to see if I follow my own advice. I try very hard to treat my students (and the various demands on their time) respectfully. I try very hard to treat my subject with appropriate respect. It matters that I explain it as well as I can and it matters that my students begin to understand it as well as they can. I always end the semester as follows: “This course comes with a warranty. If you pass this course, and later discover that you do not understand a topic that we covered, please feel free to get in touch with me for review. You should also understand that my brain is already in the process of forgetting the connection between your name and your face.”

This seems like a good place to stop.