Instructions

1. This is an opportunity for anyone who wishes to work variations of some of the exam problems (1, 2, 4b, 5a). [There are no feasible variations for 4a and 6; variations for 3 and 5b are on homework 5.] There is no “credit” involved, but I will read and comment on any solutions. In order to make things easier for me, I’d like to look at all submissions at once, so this is “due” by March 2. Also, use your own paper!

1. Let $C$ be the semi-circular contour from 3 to $3i$ along the circle of radius 3 taken counterclockwise. Compute by any correct method

$$\int_{C} (z^3 + 1 + \bar{z}) \, dz.$$ 

2. For each of the following equations, find all complex solutions $z$.

2a. $z^3 = -8$.

2b. $e^z = 5 - 5i$.

2c. $\cos z = -4$.

4b. Determine all values of $(1 + i)^i$.

5a. Find a complex number $\alpha = \rho e^{it}$ and a positive integer $n$ so that the image of region $A$ under the map $w = \alpha z^n$ is the region $B$. (Hint: first look at what happens to $z = r e^{i\theta}$ under this map.) The region $A$ is the pizza-slice-shaped region in the first quadrant, bounded by the lines $x = y$ and the positive real axis, and in the disk $|z| \leq 3$. The region $B$ is the quarter-circle in the third quadrant, bounded by $|w| \leq 4$. 