Same description as before. This is the last homework to cover material for the first exam.

1. §2.1 - 14 (The easiest proof is by induction on \( n \).)
2. §2.1 - 20 c.
3. §2.2 - 2. (Recall that \((m + 2)! = (m + 2)(m + 1)m!\).)
4. §2.2 - 8, 10.
5. Determine the image of the region

\[ A = \{(x, y) : x < 1 \text{ and } y < 1\} \]

under the map \( w = f(z) = 1/z \).

6. (E) Show that the function \( u(x, y) = y^3 - 3x^2y + 3x \) is harmonic, and calculate any harmonic conjugate \( v(x, y) \) by any correct method. Express \( f(z) = f(x + iy) := u(x, y) + i \cdot v(x, y) \) as a function of \( z \) alone.

7a. Find, carefully, all complex numbers \( z \) with the property that \( e^z = 4 + 4i \).
7b. Find, carefully, all complex numbers \( z \) with the property that \( \sin z = 10 \).

8. (E) Determine all possible value (or values) for

\[ f(z) = \log((1 - i)z) - \log(z), \]

as \( z \) ranges over the complex numbers minus the non-positive reals, and \( \log z \) denotes the Principal Value of the logarithm. For each value \( w_0 \) that you say \( f \) takes, find a specific \( z_0 \) so that \( f(z_0) = w_0 \).

9. Suppose \( f(x, y) = x^2 + y^2i \). Determine the set of \( z \) at which the Cauchy-Riemann equations are satisfied. Determine the set of \( z \) at which \( f \) is differentiable. Determine the set of \( z \) at which \( f \) is analytic.

10a. Let \( \gamma \) denote the circle \(|z| = 2\), traversed in a counter-clockwise fashion. Use the standard estimate for integrals; i.e., p.62(3), to show that

\[ \int_{\gamma} \frac{dz}{8 + 3z} \leq 2\pi. \]

10b. By considering \(|8 + 3z|^2\) separately on the semicircles in the half-planes \( x \geq 0 \) and \( x \leq 0 \), improve this estimate to

\[ \int_{\gamma} \frac{dz}{8 + 3z} \leq \frac{6}{5} \pi. \]