The last two problems are the “graduate” problems and are intended to be harder than the others. In general, problems from *Fisher* are given in text order, and problems from me are given in increasing order of difficulty. The symbol (E) means that at least part of this problem, up to variation, has appeared on one of my old Math 346 or Math 348 exams.

1. §1.1 – 6.
2. §1.1 – 18 (use induction; yes, it’s a proof!).
3. §1.2 – 2.
4. §1.2 – 16 (start with a quadratic equation, reduce it to a linear one.)
5. §1.2 – 24 and 26.
6. §1.3 – 3
7. (E) Find explicit real numbers $r$ and $s$ with the property that

$$\frac{1}{2^{173}} \left( \frac{5 + 3i}{1 + 4i} \right)^{348} = r + is.$$ 

(The fact that this was an exam problem should suggest that a judicious early computation will save a lot of work.)

8. (E) Sketch the region $R$, consisting of the complex numbers $re^{i\theta}$ with $1 \leq r \leq 2$ and $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{4}$, and also sketch its range under the mapping $w = \frac{1}{z}$.

9. §1.2 – 30.

10a. (E) Find any pair of complex numbers $(\alpha, \beta)$ with the property that if $w = \alpha z + \beta$, then the line $Im(z) = 1$ is mapped to the line $Re(w) = 2$.

10b. Find all such pairs $(\alpha, \beta)$.

General hint: start by writing $\alpha$ and $\beta$ in terms of their real and imaginary parts.

A bonus problem, for those who like number theory. I won’t grade this one, but I’ll check your work if you submit a solution.

Bonus: Define integers $a_n$ and $b_n$ by $a_n + ib_n = (3 + 4i)^n$. Prove that, for all $n \geq 1$, 

$$a_n \equiv 3 \pmod{5} \text{ and } b_n \equiv 4 \pmod{5}.$$ 

That is, there exist integers $c_n$ and $d_n$ so that 

$$a_n = 5c_n + 3 \text{ and } b_n = 5d_n + 4.$$ 

(Hint: $a_{n+1} + ib_{n+1} = (3 + 4i)(a_n + ib_n)$.)

Explain why this implies that $(3 + 4i)^n$ is never real, and hence why $\frac{\arctan 4/3}{\pi}$ is irrational.