Math 417 - Test 2 - Material for review.

1. Exam = Wed 4/13/19 in class. Closed book/notes, except for 3”x5” card. Material is class lectures, Fraleigh §9, §11, §13, §14, and homeworks 4, 5, 6, 7

2. Vocabulary - All previous group vocabulary, plus cycles, transpositions, left and right cosets of a subgroup, normal subgroups, homomorphism, isomorphism, automorphism, kernel, image, direct product of groups, the order of an element in a group, no need for groups.

3. No formulas - All previous results

4. Eq
   \[ T = \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\} \text{ as } T: 1 \rightarrow 2 \rightarrow 4 \rightarrow 1, 3 \rightarrow 3 \]
   \[ T = (2 3 4), \quad T = (2 4 3 1) \]

5. If \( T \) is a subgroup of \( S \), \( T \leq S \) if it is a normal subgroup. \( H \triangleleft S \) if \( H \) is a normal subgroup of \( S \).

6. If \( H \leq S \), the cosets of \( H \) in \( S \) form the factor group \( S/H \).

7. If \( \varphi: S \rightarrow T \) is a homomorphism, know \( \ker(\varphi) = \{ x \in S : \varphi(x) = e_T \} \) and \( \text{im}(\varphi) = \{ y \in T : \exists x \in S \text{ with } \varphi(x) = y \} \).

8. If \( \varphi: S \rightarrow T \) is isomorphic, \( S \) is isomorphic to \( T \).

9. Groups to know: \( C_n \), \( V \), \( S_3 \), \( D_4 \), \( C_m \times C_n \) \( C/\{1\} \times (\mathbb{Z}/m\mathbb{Z}) \).

10. If you know \( C \) and \( H \), you should know \( C \times H \), etc.

11. Results to know: Lagrange's Theorem. If \( H \leq S \) is normal then \( S/H \) is a group. If \( \varphi: S \rightarrow T \) is a homomorphism, then \( \ker(\varphi) \) is a normal subgroup of \( S \), and \( \text{im}(\varphi) \) is a subgroup of \( T \) that is isomorphic to \( S/\ker(\varphi) \).

12. Know how to compute the order of elements in \( C_n \), \( S_3 \), \( D_4 \), etc.

13. Induced product


15. The Sylow Theorem on classifying finite Abelian groups, and, by popular request, non-Euclidean geometry.