

Math 417 - Test 2 - Material for review.

3/29/17

1. Exam = Wed 4/13/19 in class. Closed book/notes, except for 3"x5" card. Material is class lectures, Fraleigh $\Sigma 9, 10, 11, 13, 14$. and homeworks 4, 5, 6, 7

2. Vocabulary - All previous group vocabulary, plus cycles, transpositions, left and right cosets of a subgroup, normal subgroups, homomorphism, isomorphism, automorphism, kernel, image, direct product of groups, The order of an element in a group, $\text{Ker}(\phi)$ of a group

3. No facts - All previous notation

Ex. eg $\pi = \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ Then this can be recorded as $\pi: 1 \rightarrow 2 \rightarrow 4 \rightarrow 1, 3 \rightarrow 3$ $\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$
 $\pi(1) = 2, \pi(2) = 4$
 $\pi(3) = 3, \pi(4) = 1$
 $\pi = (124)(3), \pi = (124)$

If H is a subgroup of G , $H \leq G$, if H is a normal subgroup, $H \trianglelefteq G$.
If $H \trianglelefteq G$, the cosets of H in G form the factor group G/H .

If $\phi: G \rightarrow H$ is a homomorphism, know $\text{Ker}(\phi) = \{x \in G : \phi(x) = e_H\}$
and $\text{Im}(\phi) = \{y \in H : \exists x \in G \text{ with } \phi(x) = y\}$. \leftarrow "isomorphic to".

4. Groups to know $C_n, V, S_3, D_4, C_m \times C_n \cong (\mathbb{Z}/m\mathbb{Z}) \times (\mathbb{Z}/n\mathbb{Z})$.
If you know G and H , you should know $G \times H$, etc.

5. Results to know: Lagrange's Theorem. If $H \trianglelefteq G$ is normal, then G/H is a group. If $\phi: G \rightarrow H$ is a homomorphism, then $\text{Ker}(\phi)$ is a normal subgroup of G , and $\text{Im}(\phi)$ is a subgroup of H that is isomorphic to $G/\text{Ker}(\phi)$. Know how to compute the order of elements in C_n, S_3, D_4 , etc and indirect products

6. Not on the test: Odd/even permutations, Cayley's Theorem, The basic Theorem on classifying finite Abelian groups and, by popular request, non-Euclidean geometry