Math 597 and Math 599 Course Organization, Bruce Reznick, Spring 2022

"This is gonna go on for a while." – Lou Reed

Rationale: This is the latest revision of a "Course Organization" for my graduate reading and dissertation classes; mostly descriptive of recent practice, but also partially aspirational. It is meant to serve both as a guide to my students and as a set of multiple reminders to me. It isn't motivated by any particular incident or new policy in the department or college, and is explicitly personal, applying only to me and my students. I've sent a copy to the Mathematics Graduate Office. Feedback is always welcome.

General overview: The first thing that needs to be said is that advising graduate students towards their PhD dissertation is perhaps the most important one-on-one responsibility for a tenured faculty member at a research university. These are the only places that PhDs are trained. I want to explicitly recognize its centrality to your graduate experience and future professional development. I once went sixteen years between PhD students. I cherish the opportunity to participate in your thesis. There is never a reason for a continuing graduate student to apologize for taking my time in a reading course; this is what I'm here for. (Caveat: This is not a solicitation for new students; I am close to my limit now.) At the same time, I recognize (and try to reconcile) the inevitable tension between the importance of the dissertation to the student and the power held in this process by the advisor.

Logistics1 (in person): The standard model for a reading class is a weekly 50-minute meeting. If we finish faster, that's fine. If you have a busy time in your professional life or your personal life and want to skip a week, that's fine too. In fact, it's reasonable for you to skip a couple of meetings per semester, and I won't ask why. "Load management" works in mathematics as well as sports. If you don't want to meet because you don't feel you've accomplished anything, that happens, don't worry. If it happens a couple of weeks in a row, let's at least touch base. I know a lot of techniques for handling blockages in mathematical creativity, and can tell you about the ones that I've worked on myself. (The section of my webpage called "When it comes to advice, I'm full of it" contains the semi-helpful essays "Introduction to mathematical research" and "Resources for research".)

Recent preference has been for meetings in an open classroom, usually reserved through Aaron Brewer in 273 Altgeld; he's told me that you can ask him for a room as well. (These rooms might be in adjacent buildings.) The Commons Room will soon be closed, so we might meet in my new office, 374 Altgeld. (I always felt intimidated in faculty offices when I was a student, but some students don't seem to be bothered.) Weather permitting, we can take a walk on the Quad or go for coffee. My office phone is 217-333-4284 and my home phone is 217-344-7137. It is unusual for me to be away from email during the day for more than a few hours, except when traveling, which is also the only time I carry a working cellphone. I send out a weekly circular email, usually on Saturday mornings, with notes about my schedule, my recollection of our next meeting times, upcoming seminars and other activities in the department, and a few jokes.

Logistics1 (pandemic version): If we can't meet in person, of course we can meet on Zoom, and the same comments apply as above, except for where to meet. I do not record meetings. It is especially helpful for me if you can email questions in advance, so I can TeX up answers in advance and screenshare during our regular session.

Logistics2: Self-care is always essential. Make sure you have some fun every day (no report to me needed). If you have to choose between sleep and preparing for our meeting, for goodness' sake, *please* choose sleep. No theorem is worth your health.

At some point, you will want or need me to write a letter to the Graduate Office or for a fellowship or for a job. I have a lot of practice writing letters and usually write for about 20-30 people each year overall. If you need a letter from me, don't be shy in asking. You may also have forms that need my signature. Ordinarily, if they also require a conversation, bring them to a meetings. Otherwise, leave them in my mailbox and send me an email; I'll sign and return to your mailbox.

Social media. I exist, barely, on Facebook and Twitter. I don't know what LinkedIn is really for, but I'm there too. As a general academic policy, it is awkward when students visibly "like" opinions their advisor has expressed publicly, because there's always a question about whether this has been done freely or under a sense of obligation. I won't notice or get upset if you do it or if you don't. These days, I do not follow my students, undergraduate or graduate on social media; I think that would be inappropriately intrusive.

Also, please don't forget to register for this class if you're taking it.

The work: What to read? This may be your most important decision, and it will emerge from our conversations about your interests and your goals. My goals for your thesis are almost completely determined by your goals for yourself. I know a lot of kinds of math, but you might be interested in other areas of math that I could be helpful with. *The final decision is always yours.* There are a few places I typically start (Stern sequence notes, my papers or beamer slides on my website), but as time goes on, and you tell me what resonates with you and what doesn't, the direction of our work will become clearer. Usually it takes no more than a semester for an individualization of the subject matter to take hold.

Whenever possible, theory is interspersed with specific examples working out the ideas in detail. In my experience, the first stage of mathematical discovery is the anomalous example. (According to Isaac Asimov, scientists don't say "Aha!", they say "That's funny!") Occasionally, you will start reading a paper, and decide you don't want to continue. That's fine; I know a lot of papers. Occasionally it turns out that what you're interested in is something I can't effectively advise you on, but a colleague can. If so, go for it. It's happened before. I don't take it personally. We can still say hello at tea.

If you have done some work that you want me to look at, I prefer you to send it to me the day before, and even better if you've LaTeX'd it up so I can print it out to read. (I can't read critically on a screen very well.) I will be more useful to you when I have had several passes at reading, and a chance to reflect. For you, the .tex file becomes part of a zero-th draft of your dissertation, and you can never start on *that* too early. It's helpful to put the date into the title of the file for easy retrieval. I am ruthless in red-inking drafts for content and for usage, and if you feel I've been mean, I'll show you what I do to my own drafts! Naturally, there are times when you've just proved something and are excited to talk to me about it. If you want to drop by my office, please email first so we can find a mutually convenient time.

Doing mathematics. Mathematical research is a natural extension of the homework you've done all your life in mathematics courses, except that there is no back of the book to check for the answer, and there might not even *be* a book. Research is also often a matter of synthesizing several seemingly different ideas or approaches into a new object. Your job as

a prospective researcher is to be active, not passive, in taking in mathematics: let the ideas get under your skin and visit your dreams. Be bold. No idea is so audacious that it isn't worth exploring for at least 15 minutes.

Math is hard. It has to be hard. Nobody can get a job as a professor of boiling spaghetti. Very smart people have been thinking about mathematics for a hundred generations. How dare we try to add to their understanding? I think the best way is through hard work and a deep appreciation of the incremental progress of ourselves and others. Research progress is not linear or even continuous. I've been working on some problems for more than 40 years.

One of my most important jobs is to be "present" during our meeting times and alert to the questions you are asking, both explicitly and implicitly. Every one of my students has, by the time they finished their dissertation, proved something wonderful and unexpected. That's what I'm looking for. Sergei Diaghilev, the founder of the Ballets Russes, used to say to his dancers, "Étonnez-moi." "Astonish me." Astonish me. That's why we're here.

Tricky parts: Graduate students and professors are adults who are fellow inhabitants of an intentional community known as a Department of Mathematics. When they enter into the advisor/advisee roles, things change, sometimes subtly, sometimes not. I always want to meet students where they are, shaped by their specific experiences (academic, personal, and societal), expectations and talents. But these mostly melt away when we are actively doing math together: polynomials don't care about our particularities.

One of the first books I read about education pointed out that a good teacher must "be friendly without being a friend", and so I should discuss some uncomfortable issues explicitly. The professor/student relationship is inherently asymmetric. One person must formally evaluate the other's work, and there are typically major disparities in age, experience, etc. Friends don't grade friends. Nonetheless, cordiality and mutual respect *must* prevail.

There is no algorithm for achieving a successful advisor/advisee relationship; it is an iterative process. I'll let you know how I think things are going when we work together and at the same time, *I urge you in the strongest terms to be forthright in telling me quickly about any non-optimal aspects of our professional interactions.* We both want to accelerate the convergence. (Some of you reading this have already been forthright with me. Thanks!) The easiest ways for such communications to happen are face-to-face or campus email, though in some cases, you might feel more comfortable leaving a note in my mailbox. If you ever want it, I can give you my non-departmental email address.

This brings me to the delicate and rapidly emerging issue of privilege, which many of us (though nowhere near enough) are currently working through. (Yes, my choice of the word "emerging" shows that I have enjoyed a lot of privileges.) The most pernicious thing about privilege is its invisibility to those who have it: you don't notice when things are going the way they ought to. (People without a particular privilege more easily notice when things consistently *don't* go the right way.) Privilege is an immunity[1] from some specific garbage behaviors, whose absence is easily overlooked. People who complain about an overemphasis on ethnicity or gender have probably lived the privilege of not being reminded repeatedly of how their details are different from others. As a small example, I have never been mistaken

[1]To the extent that privilege *is* immunity, I think "immunity" is the better term for effecting change. Any privilege is limited; people will simultaneously deny they have it, and fight to keep it. But immunity *spreads* and we all realize that the community is stronger when *everyone* shares in it. My personal goal is that everyone in the mathematical community should enjoy *all* its privileges.

for a custodian, office staffer, security officer or food server at a conference, nor given the stink-eye as a suspected interloper during refreshment breaks. It is an essential exercise to reflect periodically on one's own privileges. I keep finding new ones.

Like power, privilege is systemically asymmetric in our interactions: only one of us looks like the professor in a cartoon. And one of my many privileges over the years is that I haven't had to overcome demographic bigotry to establish myself as a plausible authority figure. (To be fair, I've had to deal with my own temperamental discomfort with the professional necessity of *being* an authority figure.) If privilege issues come up as we work together (bad assumptions, unintended affronts, etc.) please let me know as soon as possible, so I can fix what I'm doing. It is not your job to educate me in general, but I appreciate being informed when it is glaringly obvious that I don't know something I should.

One final tricky point about me. I will probably be retiring in the next few years. However, there is a long tradition in this department of emeritus faculty giving reading courses and advising PhD dissertations, and I hope to continue participating for a long time.

Your instructor: This section contains information for you and for anyone who asks about me. My CV is accessible from my webpage. I am usually an experimental pure mathematician. My work is defined by particular problems I find interesting, and I find it important to learn the history and literature of these problems while I'm exploring. (Much wonderful mathematics has been "reclaimed by the underbrush" and needs to be rediscovered.) I am not a master of any general area, and my interests are not usually motivated by the current socio-professional consensus of the area's "important" questions.

To paraphrase from my (often unsuccessful) grant applications: "My research is usually classified in 11 by MathSciNet, but my conferences are in 14. The work has recently been harvested in the delightful valley bounded by number theory, algebraic geometry and analysis, under a strongly combinatorial breeze." My main research these days has involved the representation of homogeneous polynomials (forms) as a sum of higher powers of forms, whether as sums of real squares (Hilbert's 17th problem), as linear combinations of higher powers of linear forms over various fields (Waring representations) or as sums of powers of complex quadratic forms (recent results in 19th century mathematics). I have also worked on a variety of digital problems in combinatorial number theory, often emanating from properties of the Stern sequence, and from the pattern of the configurations of lattice points in polytopes, with applications to sums of squares. (My own PhD was in functional analysis, leading me to Hilbert's 17th Problem and the higher degree Waring representation problem over $\mathbb{R}$.) My papers might or might not match the current collective identification of "hot" topics, but I will never suggest that you work on a narrow thesis topic that only you and I would care about.

Finally, I need to discuss another rather taboo but very important subject: my answers to the question "What does mathematics mean to you?". These are presented in part to help you develop your own answers. There's no right or wrong; the answer varies from person to person and for one person, may change over time. Here's my January 2022 version.

"Euclid alone has looked on Beauty bare." – Edna St. Vincent Millay

Mathematics fills the spiritual niche in my life. An organized religion offers, I think, three things: (i) an explanatory story of the world, leading to (ii) a set of rules about how humans should interact, creating (iii) a community of people who share (i) and (ii). Mathematics

is good with (i), great with (iii), but is basically silent about (ii). The Golden Rule is a symmetry principle, but not too much else in (ii) is axiomatic. I plug the gap with the (regrettably, still aspirational) intention to treat everyone I meet with respect and kindness, with the understanding that all people are equally important. Much of what I've written here can be considered an explication of this last point: professors are neither more (nor less) important than students.

There are many different manifestations of mathematical transcendence: holy places (e.g. the Altgeld Library, Oberwolfach, etc.), holy texts (e.g. Bourbaki, Ramanujan's notebooks, etc.), holy objects (e.g. q-series, symplectic manifolds, etc.), holy incantations (e.g. Euclid's proofs that $\sqrt{2}$ is irrational and that there are infinitely many primes, etc.), holy priestly lineages (e.g. a PhD student of a PhD student of a PhD student of Bill Thurston, having Erdös number one, etc.), and so on. In this extended metaphor, I find myself to be a kind of pantheist: seeing mathematical truth as something bigger than myself which manifests itself everywhere in ordinary life. My ideal is to look at the mathematical objects themselves, in as direct and unmediated a way as I can. That's happened a few times, and it's a wonderful feeling. But it's usually impossible. Like all mathematicians, I've stood on the shoulders of many giants in everything I've accomplished. Let me repeat that your mileage will vary on this, and there are no unique solutions. No disrespect has been intended in these paragraphs to people with different beliefs.

Acknowledgments: Over the last few decades, I have worked with and learned from a group of very patient and talented graduate students. I'm happy that I am still in regular contact with my completed students, hopefully as friends.

Given what I said earlier, you might be curious about how students become friends. The most important items an advisor signs in the student's last year are (of course) the dissertation forms and the Letter of Recommendation. Unless you get a postdoc and then apply for a tenure-track position, this might well be the last recommendation letter I write for you. It is important after the PhD for you to achieve intellectual "separation" from your advisor, if you haven't already done so earlier. Find other mathematical interests beyond your dissertation and my research topics. And then, when we talk, it will be on a more equal footing, as colleagues and friends.

I want to recognize my students here, in roughly chronological order (I wouldn't have known how to be an advisor, or write these notes, without their continuing and helpful input): (completed) Julie Simon, William Harris, Ricardo Rojas, Han Duong, Melissa Dennison, Supawadee Prugsapitak, Milos Curcic, Jennifer Lansing, Katie Anders, Wipawee Tangjai, Neriman Tokcan, Sakulbuth Ekvittayaniphon, Simone Sisneros-Thiry; (current PhD or reading students) Ben Wright, Dana Neidinger, Grace Jaffe, David Altizio, Yansy Perez, Qiuling Fan.

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Final word: "Go out there and prove some beautiful, unexpected, astonishing, and true theorems!" – BR 5/10/19, 7/27/19, 12/16/19, 8/16/20, 8/26/21, 1/8/22