Math 597 and Math 599 Course Organization, Bruce Reznick, Fall 2019

"This is gonna go on for a while." – Lou Reed
"About a page longer than last time." –BR

Rationale: This is a "Course Organization" for my graduate reading and dissertation classes; mostly descriptive and codifying recent practice, but also partially aspirational. It is meant to serve both as a guide to my students and as a set of multiple reminders to their instructor. It isn't motivated by any particular incident or new policy in the department or college, and is explicitly personal, applying only to me and my students. I've sent a copy to the Mathematics Graduate Office. Feedback is always welcome.

General overview: The first thing that needs to be said is that advising graduate students towards their PhD dissertation is perhaps the most important one-on-one responsibility for a tenured faculty member at a research university. (These are the only places that PhDs are trained.) I want to explicitly recognize its centrality to your graduate experience and future professional development. I once went sixteen years between PhD students. I cherish the opportunity to participate in your thesis. There is never a reason for a continuing graduate student to apologize for taking my time in a reading course; this is what I'm here for. (Caveat: This is not a solicitation for new students; I am close to my limit now.)

Logistics: The standard model for a reading class is a weekly 50-minute meeting. If we finish faster, that's fine. If you have a busy time in your professional life or your personal life and want to skip a week, that's fine too. If you don't want to meet because you don't feel you've accomplished anything, that happens, don't worry. If it happens a couple of weeks in a row, let's at least touch base. I know a lot of techniques for handling blockages in mathematical creativity, and can tell you about the ones that I've worked on myself. (The section of my webpage called "When it comes to advice, I'm full of it" contains the semi-helpful essays "Introduction to mathematical research" and "Resources for research".)

The generic place for our meetings is the Commons Room, 321 Altgeld, rather than my horribly disorganized office, 327 Altgeld. (Also, I always felt intimidated in faculty offices when I was a student.) We can also look for an open classroom, or ask Aaron Brewer in 273 Altgeld to reserve us a room on a one-time or recurring basis. (He's told me that you can ask him for a room as well.) If you'd rather meet in a non-public room, and no classroom is available, 327 is there, or weather permitting, we can take a walk on the Quad. My office phone is 217-333-4284 and my home phone is 217-344-7137. It is unusual for me to be out of email contact during the day for more than a few hours, except when traveling, which is also the only time I carry a working cellphone. I send out a weekly circular email, usually on Saturday mornings, with notes about my schedule, my recollection of our meeting times, and upcoming seminars and other activities in the department.

Self-care is always essential. Make sure you have some fun every day (no report to me needed). If you have to choose between sleep and preparing for our meeting, for goodness' sake, *please* choose sleep. No theorem is worth your health.

The work: What to read? This may be your most important decision, and it will emerge from our conversations about your interests and your immediate and long-term goals. (I know a lot of kinds of math, but you might be interested in other areas of math that I could be helpful with.) *The final decision is always yours.* There are a few places I typically start (Stern sequence notes, my papers or beamer slides on my website), but as time goes on, and you tell me what resonates with you and what doesn't, the direction of our work will become clearer. Usually it takes no more than a semester for an individualization of the subject matter to take hold.

Whenever possible, the theory is interspersed with specific examples or problems working out the ideas in detail. In my experience, the first stage of mathematical discovery is the anomalous example. (According to Isaac Asimov, scientists don't say "Aha!", they say "That's funny!") Occasionally, you will start reading a paper, and decide you don't want to continue. That's fine; I know a lot of papers. Occasionally it turns out that what you're interested in is something I can't effectively advise you on, but a colleague can. If so, go for it. It's happened before. I don't take it personally. We can still say hello over cookies.

If you have done some mathematics that you want me to look at, it's better for me if you can get it to me the day before, and even better if you've LaTeX'd it up so I can print it out to read. (I can't read critically on a screen very well.) I am also more effective when I have several passes at reading, and a chance to think about relevant references. For you, the .tex file becomes part of a zero-th draft of your dissertation, and you can never start on that too early. I find it helpful to put the date into the title of the file for easy retrieval. I am ruthless in red-inking drafts for content and for usage, and if you feel bad about that, I'll show you what I do to my own drafts! Naturally, there are times when you've just proved something and are excited to talk to me about it. If you want to drop by my office, please email first so we can find a mutually convenient time.

Doing mathematics. Mathematical research is a natural extension of the homework you've done all your life in mathematics courses, except that there is no back of the book to look for the answer, and there might not even *be* a book. Research is also often a matter of synthesizing several seemingly different ideas or approaches into a new object. Your job as a prospective researcher is to be active, not passive, in taking in mathematics: let the ideas get under your skin and visit your dreams.

Be bold. No idea is so wild that it isn't worth exploring for at least 15 minutes. Barbie was right: "Math *is* hard." It has to be hard. Very smart people have been thinking about mathematics for a hundred generations. How dare we try to add to their understanding? I think the best way is through hard work and a deep appreciation of the incremental progress of ourselves and others.

One of my most important jobs is to be constantly "present" during our meeting times and alert to the questions you are asking, both explicitly and implicitly. Every one of my students has, by the time they finished their dissertation, proved something wonderful and unexpected. That's what I'm looking for. Google tells me that Sergei Diaghilev, the founder of the Ballets Russes, used to say to his dancers, "Étonnez-moi." "Astonish me." Astonish me. That's why we're here.

Secondary logistics: At some point, you will want or need me to write a letter to the Graduate Office or for a fellowship or for a job. I have a lot of practice writing letters for undergrads and everyone else, and usually write each year for 20-30 people overall. (I have had a decent success rate, but there are no guarantees.) If you need a letter from me, don't be shy in asking. You may also have forms that need my signature. Ordinarily, if they also require a conversation, bring them to our meetings (extra ones if needed). Otherwise, leave them in my mailbox and send me an email; I'll sign and return to your mailbox.

Social media. I exist, barely, on Facebook and Twitter. I don't know what LinkedIn is really for, but I'm there too. As a general academic policy, it is awkward when students visibly "like" opinions their advisor has expressed publicly, because there's always a question about whether this has been done "freely" or under a sense of obligation. I won't notice or get upset if you do it or if you don't. These days, I do not follow my students, undergraduate or graduate on social media; I think that would be inappropriately intrusive.

Please don't forget to register for the reading class if you're taking it. Petitions work, but they are tedious.

Tricky parts: Graduate students and professors are adults who are fellow inhabitants of an intentional community known as a Department of Mathematics. When they enter into the advisor/advisee roles, things change, sometimes subtly, sometimes not. I always want to meet students where they are, shaped by their specific experiences (academic and societal), expectations and talents. But these mostly melt away when we do math together: polynomials don't care about our particularities.

One of the first books I read about education pointed out that a good teacher must "be friendly without being a friend", and so I should discuss some uncomfortable issues explicitly. The professor/student relationship is inherently asymmetric. One person must formally evaluate the other's work, and there are typically major disparities in age, experience, etc. Friends don't grade friends. Nonetheless, cordiality and mutual respect must prevail.

There is no algorithm for achieving a successful advisor/student relationship; it is an iterative process. I'll let you know how I think things are going when we work together and at the same time, *I urge you in the strongest terms to be forthright in telling me quickly about any non-optimal aspects of our professional interactions.* We both want to accelerate the convergence. (Some of you reading this have already been forthright with me. Thanks!) The easiest ways for such communications to happen are face-to-face or campus email, though in some cases, you might feel more comfortable leaving a note in my mailbox. If you ever want it, I can give you my non-departmental email address.

Somewhat related to this is the delicate and rapidly emerging issue of privilege, which I and many others (though nowhere near enough) are currently working through. The most pernicious thing about privilege is its invisibility: you don't notice when things are going the way they should. (People without a particular privilege more easily notice when things consistently *don't* go the right way.) Privilege is an immunity[1] from some specific garbage behaviors, whose absence is easily overlooked. People who complain about an overemphasis

[1]To the extent that privilege *is* immunity, I think "immunity" is the better term for effecting change. Any privilege is limited; people will simultaneously deny they have it, and fight to keep it. But immunity *spreads* and we all realize that the community is stronger when *everyone* shares in it.

on ethnicity or gender have probably lived the privilege of not being reminded repeatedly of how their details are different from others. For example, I have never been mistaken for a custodian, secretary or food server at a conference.

Like power, aspects of privilege will be systemically asymmetric in our interactions: only one of us looks like the professor in a cartoon. And one of my many privileges over the years is that I haven't had to overcome demographic bigotry to establish myself as a plausible authority figure. (To be fair, I've had to deal with my own temperamental discomfort in the professional necessity of *being* an authority figure.) If privilege issues come up as we work together (bad assumptions, unintended affronts, etc.) please let me know as soon as possible, so I can fix what I'm doing. It is not your job to educate me in general, but I appreciate being told when it is glaringly obvious that I don't know something I should.

Your instructor: This section contains information for you and for anyone who asks about me. My CV is accessible from my webpage. I am usually an experimental pure mathematician. My work is defined by particular problems I find interesting, and I find it important to learn the history and literature of these problems while I'm exploring. (Much wonderful mathematics has been "reclaimed by the underbrush" and needs to be rediscovered.) I am not a master of any general area, and my interests are not usually motivated by the current social consensus of the area's "important" questions.

To paraphrase from my grant applications (I often don't get the grants): "My research is usually classified in 11 by MathSciNet, but my conferences are in 14. The work has recently been harvested in the delightful valley bounded by number theory, algebraic geometry and analysis, under a strongly combinatorial breeze." Recently, my main research has involved the representation of homogeneous polynomials (forms) as a sum of higher powers of forms, whether as sums of real squares (Hilbert's 17th problem), as linear combinations of higher powers of linear forms over various fields (Waring representations) or as sums of powers of complex quadratic forms (recent results in 19th century mathematics). I have also worked on a variety of digital problems in combinatorial number theory, often emanating from properties of the Stern sequence, and from the pattern of the configurations of lattice points in polytopes. (My own PhD was in functional analysis, in a problem that ultimately led me to Hilbert's 17th problem and to the Waring representation problem over $\mathbb{R}$ with non-negative coefficients.) My papers might or might not match the current collective identification of "hot" topics, but I will never propose that you work on a narrow thesis topic that only you and I would care about.

Finally, I want to discuss another rather taboo but very important subject. These are my answers to the question "What does mathematics mean to you?", and are presented in service of helping you develop your own answers. There's no right or wrong; the answer varies from person to person and for one person, may change over time. Here's my July 2019 version.

"Euclid alone has looked on Beauty bare." – Edna St. Vincent Millay

Mathematics fills the spiritual niche in my life. An organized religion offers, I think, three things: (i) an explanatory story of the world, leading to (ii) a set of rules about how humans should behave, creating (iii) a community of people who share (i) and (ii). Mathematics

is good with (i), great with (iii), but is basically silent about (ii). The Golden Rule is a symmetry principle, but not too much else in (ii) is axiomatic. I plug the gap with the (regrettably, still aspirational) desire to treat everyone I meet with respect and kindness, with the understanding that we are all equally important.

There are many different manifestations of mathematical transcendence: as holy places (e.g. the Altgeld Library, Oberwolfach, etc.), holy texts (e.g. Bourbaki, Ramanujan's notebooks, etc.), holy objects (e.g. q-series, symplectic manifolds, etc.), holy incantations (e.g. Euclid's proofs that $\sqrt{2}$ is irrational and that there are infinitely many primes, etc.), holy priestly lineages (e.g. a PhD student of a PhD student of a PhD student of Bill Thurston, having Erdös number one, etc.) and so on. In this extended metaphor, I find myself to be a kind of pantheist: seeing mathematical truth as something bigger than myself which manifests itself everywhere in ordinary life. My ideal is to look at the mathematical objects themselves, in as direct and unmediated a way as I can. That's happened a few times, and it's a wonderful feeling. But it's usually impossible. Like all mathematicians, I've stood on the shoulders of many giants in everything I've accomplished. Let me repeat that your mileage will vary on this, and there are no unique solutions; no disrespect has been intended in these paragraphs to people with different beliefs.

Acknowledgments: Much of what is written here is the result of working for thirty plus years with a long series of very patient and talented graduate students; I'm happy that I am still in regular contact with my completed students, hopefully as friends.

Given what I said earlier, you might be curious about how students become friends! The most important items an advisor signs in the student's last year are the dissertation forms (duh) and the letter of recommendation. Unless you get a postdoc and then apply for a tenure-track position, this might well be the last recommendation letter I write for you. It is important after the PhD for you to achieve intellectual "separation" from your advisor. It's good for you to find other mathematical interests beyond your dissertation and my specific research topics. And then, when we talk, it will now be on a more equal footing.

I want to recognize my students here, in roughly chronological order (I wouldn't have known how to be an advisor without their continuing and helpful input): (completed) Julie Simon, William Harris, Ricardo Rojas, Han Duong, Supawadee Prugsapitak, Melissa Dennison, Milos Curcic, Jennifer Lansing, Katie Anders, Wipawee Tangjai, Neriman Tokcan, Sakulbuth Ekvittayaniphon; (current) Ben Wright, Simone Sisneros-Thiry, Dana Neidinger, Grace Jaffe, Nicole Yamzon, Dania Morales.

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Final word: "Go out there and prove some beautiful, unexpected, astonishing, and true theorems!" – BR 5/10/19, 7/27/19.