

## Math 597 and Math 599 Course Organization, Bruce Reznick, Summer 2019

“This is gonna go on for a while.” – Lou Reed

**Rationale:** I’ve decided to write a “Course Organization” for my graduate reading and dissertation classes; mostly descriptive, but partially aspirational. It mostly codifies recent practice, although some of the stuff at the end is new. This is not motivated by any particular incident or new policy in the department or college. And it is strictly personal, not prescriptive to any colleague: it applies only to me. I’ll be sending a copy to the Graduate Office, but they haven’t actually approved it. From now on, I plan to distribute the evolving version of this document each term. Your feedback is always welcome.

**General overview:** The first thing that needs to be said is that advising graduate students towards their PhD dissertation is perhaps the most important one-on-one responsibility for a tenured faculty member at a research university. (These are the only places that PhDs are trained.) I want to explicitly recognize its centrality to your graduate experience and future professional development. I once went sixteen years between PhD students. I cherish the opportunity to participate in your thesis. There is never a reason for a continuing graduate student to apologize for taking my time in a reading course; this is what I’m here for. (Caveat: This is not a solicitation for new students; I may be close to my limit now.)

**Logistics:** The standard model for a reading class is a weekly 50-minute meeting. If we finish faster, that’s fine. If you have a busy time in your professional life or your personal life and want to skip a week, that’s fine too. If you don’t want to meet because you don’t feel you’ve accomplished anything, that happens, don’t worry. If it happens a couple of weeks in a row, let’s at least touch base. I know a lot of techniques for handling blockages in mathematical creativity, and can tell you about the ones that I’ve worked on myself. (The section of my webpage called “When it comes to advice, I’m full of it” contains the semi-helpful essays “Introduction to mathematical research” and “Resources for research”.)

The generic place for our meetings is the Commons Room, 321 Altgeld, rather than my horribly disorganized office, 327 Altgeld. (I felt intimidated in faculty offices when I was a student.) We can also look for an open classroom, or ask Aaron Brewer in 273 Altgeld to reserve us a room on a one-time or recurring basis. (He’s told me that you can ask him for a room as well.) If you’d rather meet in a non-public room, and no classroom is available, 327 is there, or weather permitting, we can take a walk on the Quad. My office phone is 333-4284 and my home phone is 344-7137. I have screens in my office and house and it is unusual for me to be out of email contact during the day for more than a few hours, except when traveling, which is also the only time I carry a cell (ancient and not wi-fi). I send out a weekly circular email, usually on Saturday mornings, with notes about my schedule, my understanding of our meeting times and upcoming seminars and talks in the department.

**The work:** What to read? This may be your most important decision, and it will emerge from our conversations about your interests and your immediate and long-term goals. (I know some things, but you might be interested in other things that I can be helpful with.) *The final decision is always yours.* There are a few places I typically start (Stern sequence

notes, my papers or beamer slides on my website), but as time goes on, and you tell me what resonates with you and what doesn't, the direction of our work will become clearer. Usually it takes no more than a semester for an individualization of the subject matter to take hold.

Whenever possible, this is interspersed with specific examples or problems working out the ideas in detail. In my experience, the first stage of mathematical discovery is the anomalous example. (According to Isaac Asimov, scientists don't say "Aha!", they say "That's funny!") Occasionally, you will start reading a paper, and decide you don't want to continue. That's fine; I know a lot of papers. Occasionally it turns out that what you're interested in something I can't effectively advise you on, but a colleague can. If so, go for it. It's happened before. I don't take it personally. We can still say hello over cookies.

If you have done some mathematics that you want me to look at, it's better for me if you can get it to me the night before, and even better if you've LaTeX'd it up so I can print it out to read. I can't read critically on a screen very well. I am also more effective when I have several passes at reading, and a chance to think about relevant references. For you, the notes become part of a zero-th draft of your dissertation, and you can never start on that too early. I find it helpful to put the date into the title of the .tex file for easy retrieval. I am ruthless in red-inking drafts for content and for usage, and if you feel bad about that, I'll show you what I do to my own drafts! Naturally, there are times when you've just proved something and are excited to talk to me about it. If you want to drop by my office, please email first so we can find a mutually convenient time.

Doing mathematics. Mathematical research is a natural extension of the homework you've done all your life in mathematics courses, except that there is no back of the book to look for the answer, and there might not even *be* a book. Research is also often a matter of synthesizing several seemingly different ideas or approaches into a new object. Your job as a prospective researcher is to be active, not passive in taking in mathematics: let the ideas get under your skin and visit your dreams.

One of my most important jobs is to be constantly "present" during our meeting times and alert to the questions you are asking, both explicitly and implicitly. Every one of my students has, by the time they finished their dissertation, proved something wonderful and unexpected. That's what I'm looking for. Google tells me that Sergei Diaghilev, the founder of the Ballets Russes, used to say to his dancers, "Étonnez-moi." "Astonish me." Astonish me. That's why we're here.

**Secondary logistics:** At some point, you will want or need me to write a letter to the Graduate Office or for a fellowship. That's also part of my job. I have a lot of practice writing letters for undergrads and everyone else, and usually write for 20-30 people a year. (I have a decent success rate, but there are no guarantees.) If you need a letter from me, don't be shy in asking. You may also have forms that need my signature. Ordinarily, if they also require a conversation, bring them to our meetings (extra ones if needed). Otherwise, leave them in my mailbox and send me an email; I'll sign and return to your mailbox.

Social media. I exist, barely, on Facebook and Twitter. I don't know what LinkedIn is really for, but I'm there too. As a general academic policy, it is awkward when students visibly "like" opinions their advisor has expressed publicly, because there's always a question about whether this has been done "freely" or under a sense of obligation. I won't get upset

if you do it, and I won't notice if you don't. These days, I do not follow my students, undergraduate or graduate. I think that would be inappropriately intrusive.

**Tricky parts:** One of the first books I read about education pointed out that a good teacher must “be friendly without being a friend”, and this makes me want to discuss some uncomfortable issues explicitly. The general professor/student relationship is inherently asymmetric. One person must formally evaluate the other's work, and there are typically major disparities in age, experience, etc. Friends don't grade friends.

There is no algorithm for achieving a successful advisor/student relationship; it is an iterative process. I'll let you know how I think things are going when we work together and at the same time, *I urge you in the strongest terms to be forthright in telling me quickly about any non-optimal aspects of our professional interactions.* We both want to accelerate the convergence. (Some of you reading this have already been forthright with me. Thanks!) The easiest ways for such communications to happen are face-to-face or campus email, though in some cases, you might feel more comfortable leaving a note in my mailbox. If you ever want it, I can provide my non-departmental email address.

Somewhat related to this is the delicate and rapidly emerging issue of privilege, which I and many others (though nowhere near enough) are currently working through. The most pernicious thing about having privilege is its invisibility: it's hard to notice when things are going the way they should. (People without a particular privilege more easily notice when things consistently *don't* go the right way.) Privilege is an immunity from some specific garbage behaviors, whose absence is easily overlooked. Again, this is asymmetric in our interactions: only one of us looks like the professor on the beer label. One of my many privileges over the years is that I haven't had to fight demographic bigotry to establish myself as a plausible authority figure. (To be fair, I've had to fight my own temperamental discomfort in asymmetric relationships.) If privilege issues come up as we work together (bad assumptions, micro-aggressions, etc.) please let me know as soon as possible, so I can try to fix what I'm doing.

**Your instructor:** This section contains information for you and for anyone who asks about me. My CV is accessible from my webpage. I am usually an experimental pure mathematician. To paraphrase from my grant applications (I often don't get the grants): “My research is usually classified in 11 by MathSciNet, but my conferences are in 14. The work has recently been harvested in the delightful valley bounded by number theory, algebraic geometry and analysis, under a strongly combinatorial breeze.” Recently, my main research has involved the representation of homogeneous polynomials (forms) as a sum of higher powers of forms, whether as sums of real squares (Hilbert's 17th problem), as linear combinations of higher powers of linear forms over various fields (Waring representations) or as sums of powers of complex quadratic forms (recent results in 19th century mathematics). I have also worked on a variety of digital problems in combinatorial number theory, often emanating from properties of the Stern sequence, and from the pattern of the configurations of lattice points in polytopes. (My own PhD was in functional analysis, in a problem that ultimately led me to Hilbert's 17th problem and to the Waring representation problem over  $\mathbb{R}$  with non-negative coefficients.) My papers may or may not match the current collective understanding of “hot”

topics, but I'll never propose that you work on a narrow thesis topic that only you and I would care about.

Finally, I want to discuss a rather taboo but very important subject. These are my answers to the question "What does mathematics mean to you?", and are presented in service of helping you develop your own answers. There's no right or wrong; the answer varies from person to person and for one person, may change over time. Here's my May 2019 version.

"Euclid alone has looked on Beauty bare." – Edna St. Vincent Millay

Mathematics fills the spiritual niche in my life. An organized religion offers, I think, three things: (i) an explanatory story of the world, leading to (ii) a set of rules about how humans should behave, creating (iii) a community of people who share (i) and (ii). Mathematics is good with (i), great with (iii), but is badly negligent about (ii). I plug the gap with the (regrettably, still aspirational) desire to treat everyone I meet with respect and kindness. The Golden Rule is a symmetry principle.

There are many different manifestations of mathematical transcendence: as holy places (e.g. Cambridge, the Altgeld Library, Oberwolfach, etc.), holy texts (e.g. Bourbaki, Ramanujan's notebooks, etc.), holy objects (e.g.  $q$ -series, symplectic manifolds, the Axiom of Choice, etc.), holy priestly lineages (e.g. a PhD student of a PhD student of a PhD student of Bill Thurston, having Erdős number one, etc.) and so on. In this extended metaphor, I find myself to be a kind of pantheist: seeing mathematical truth as something bigger than myself which manifests itself everywhere in ordinary life. My ideal is to look at the mathematical objects themselves, in as direct and unmediated a way as I can. That's happened a few times, and it's a wonderful feeling. But it's usually impossible. Like all mathematicians, I've stood on the shoulders of many giants in everything I've accomplished. Let me repeat that your mileage will vary on this, and there are no unique solutions; no disrespect has been intended in these paragraphs to people with different beliefs.

**Acknowledgments:** Much of what is written here has been developed by working for the last thirty plus years with a long series of very patient and graduate students; I'm happy that I am still in regular contact with my completed students, hopefully as friends. I want to recognize them here, in roughly chronological order: (completed) Julie Simon, William Harris, Ricardo Rojas, Han Duong, Supawadee Prugsapitak, Melissa Dennison, Milos Curcic, Jennifer Lansing, Katie Anders, Wipawee Tangjai, Neriman Tokcan, Sakulbuth Ekvittayaniphon; (current) Ben Wright, Simone Sisneros-Thiry, Dana Neidinger and Grace Jaffe. I wouldn't have known how to be an advisor without your continuing and helpful input.

**Final word:** "Go out there and prove some beautiful, unexpected and true theorems!" – BR 5/10/19.