1.7. **Exercises.** Do some of these. Extensions of Problem 10 will show up in subsequent exercise sets, so it's a good one to do. Let's say a deadline of Fri. Feb. 3, at the beginning of class, at which point I'll pass out solutions.

1. Write today's date as MMDDYYYY $\in [10^7, 10^9)$ and compute $s(n)$. For example, the first day of class was Jan. 18, 2012, and $s(01182012) = 1244$. (Europeans should use DDMMYYYY; $s(18012012) = 15394$.) This problem can be done on several days in a row, especially using a program.

2. Determine $n$ so that $s(n) = 2012$ and $s(n+1) = 595$. Note that $2012 = 2^2 \cdot 503$ and $595 = 5 \cdot 7 \cdot 17$ are relatively prime.

3. Prove that

$$\sum_{n=0}^{N}{}^{*} n = \frac{N^2}{2}, \qquad \sum_{n=0}^{N}{}^{*} n^2 = \frac{N^3}{3} + \frac{N}{6},$$

and compute

$$\sum_{n=0}^{N}{}^{*} n^3.$$

4. Let $\nu_p(n)$ denote the exponent of $p$ in the prime factorization of $n$. Show that

$$\frac{s(n-1) + s(n+1)}{s(n)} = 1 + 2\nu_2(n).$$

5. Determine, by any correct method, all odd integers $n$ so that $s(n) \in \{10, 11\}$.

6. Using (1.22), compute $[n_r]_2$; there are two slightly different answers, depending on the parity of $r$.

7. Find and prove a formula relating

$$T(n) := \sum_{k=0}^{\lfloor n/3 \rfloor} s(n-3k)$$

and $S(n)$.

8. For $r \geq 1$ and $t \geq 0$, compute $s((2^r+1)^2)$ and $s((2^r-1)(2^{r+t}-1))$.

9. Sometimes the Stern sequence fakes you out. Suppose

$$c_r = \left(\sum_{n \in I_r}{}^{*} 1\right)\left(\sum_{n \in I_r}{}^{*} s(n)^2\right) - \left(\sum_{n \in I_r} s(n)\right)^2.$$

We know that $c_r \geq 0$ by the Cauchy-Schwarz Inequatilty. Show that $c_1 = 1$, $c_2 = 11$ and $c_3 = 111$. Compute the disheartening value of $c_4$.

10. (The first in a series.) Show that for $k \in \mathbb{N}$, there exist functions $A(k), B(k)$ so that for $r > \log_2 k$,

$$s(2^r - k) = A(k)r + B(k).$$

The most instructive way to do this problem is to see what happens for small values of $k$ first. The recursion is helpful, the continued fraction, less so.