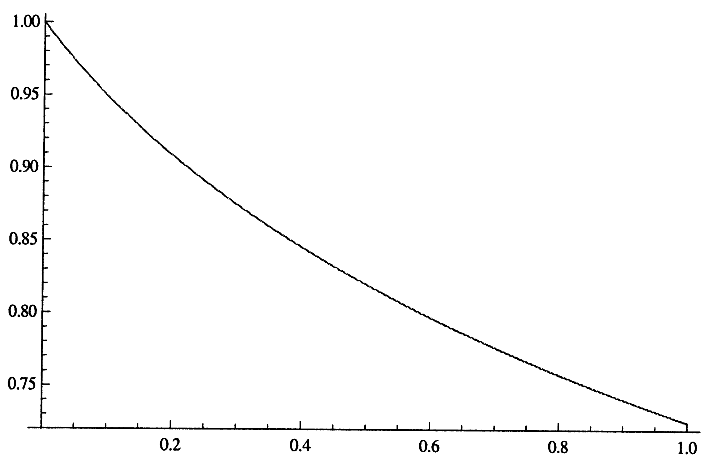
```
In[1]:= f[x_, n_] := Product[(1 + x^(2^k) + x^(2^(k+1))), {k, 0, n}]

In[4]:= t[m_] := 2^(-1/2^m)

In[6]:= Table[{n, N[t[n]]}, {n, 6, 20}]

Out[6]= {{6, 0.989228}, {7, 0.994599}, {8, 0.997296},
  {9, 0.998647}, {10, 0.999323}, {11, 0.999662},
  {12, 0.999831}, {13, 0.999915}, {14, 0.999958},
  {15, 0.999979}, {16, 0.999989}, {17, 0.999995},
  {18, 0.999997}, {19, 0.999999}, {20, 0.999999}}

In[7]:= ga = Log[3] / Log[2];

In[8]:= {ga, N[ga]}
```

Out[8]= $\left\{\frac{\text{Log}[3]}{\text{Log}[2]}, 1.58496\right\}$

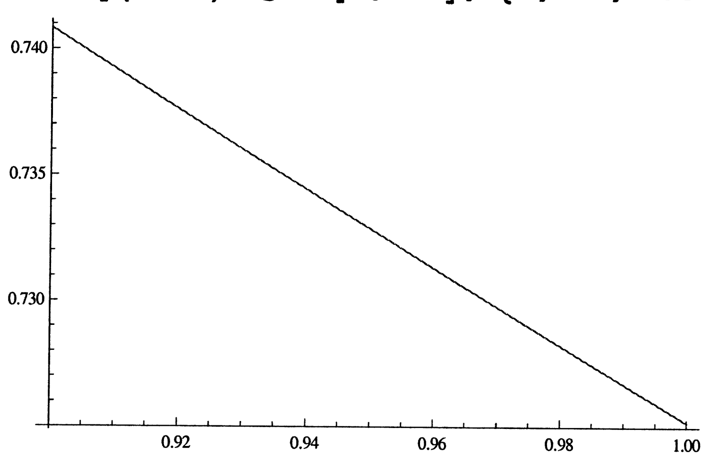
```
In[12]:= Plot[(1 - x)^ga f[x, 25], {x, 0, .99999}]
```

Out[12]=




```
In[13]:= Plot[(1 - x)^ga f[x, 25], {x, .9, .99999}]
```

Out[13]=



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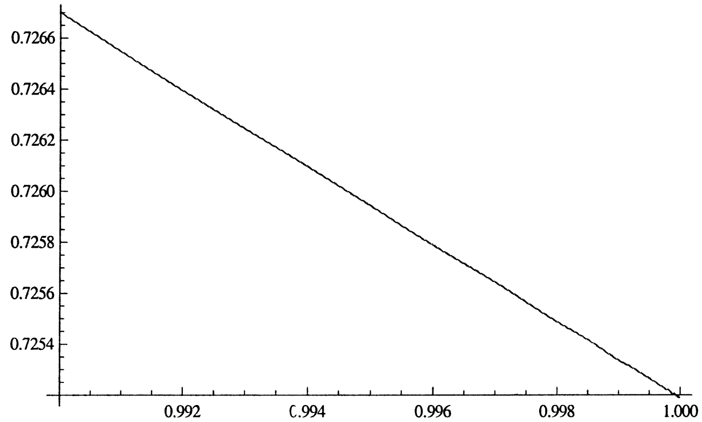
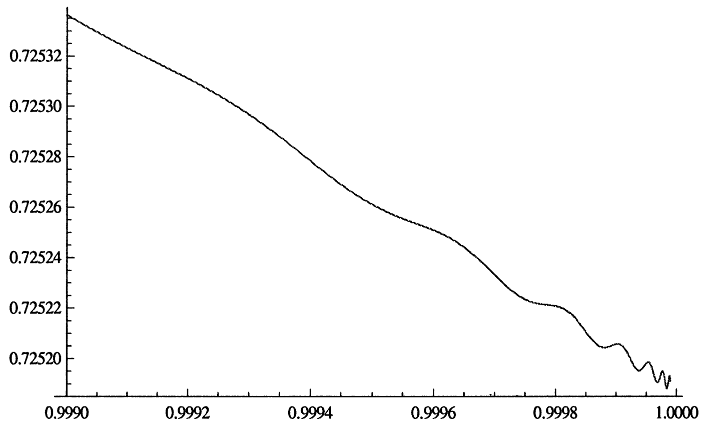
In[14]:= `Plot[(1 - x) ^ga f[x, 25], {x, .99, .99999}]`

Out[14]=



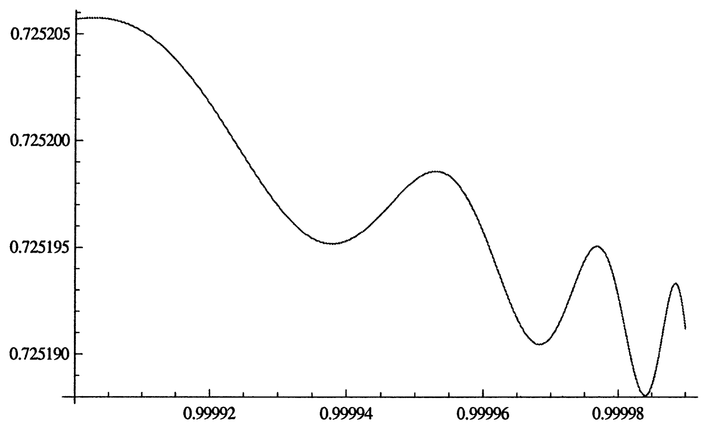
In[15]:= `Plot[(1 - x) ^ga f[x, 25], {x, .999, .99999}]`

Out[15]=



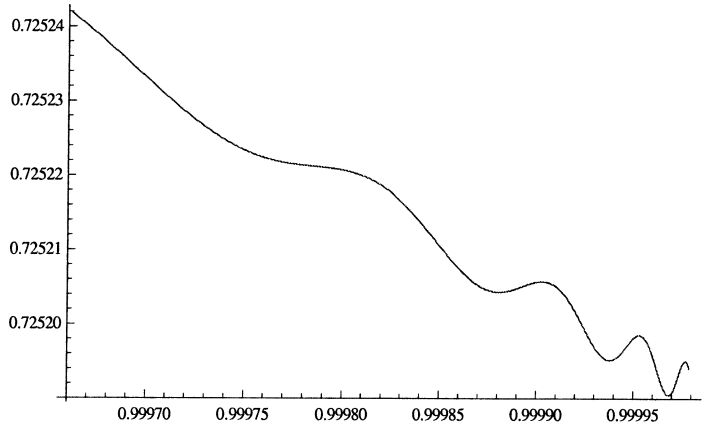
In[16]:= `Plot[(1 - x) ^ga f[x, 25], {x, .9999, .99999}]`

Out[16]=



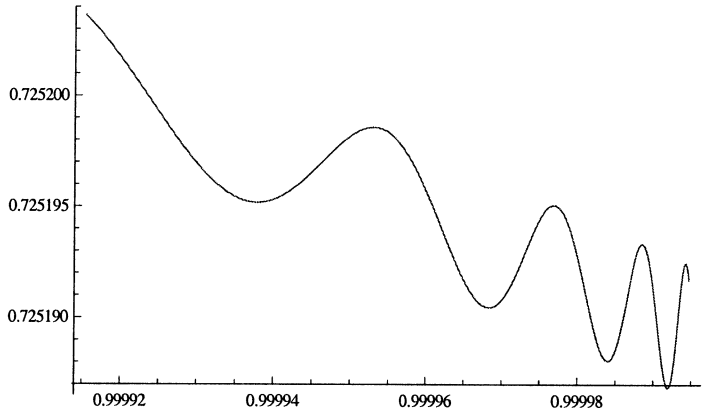
```
In[17]:= Plot[(1 - x)^ga f[x, 25], {x, t[11], t[15]}]
```

Out[17]=



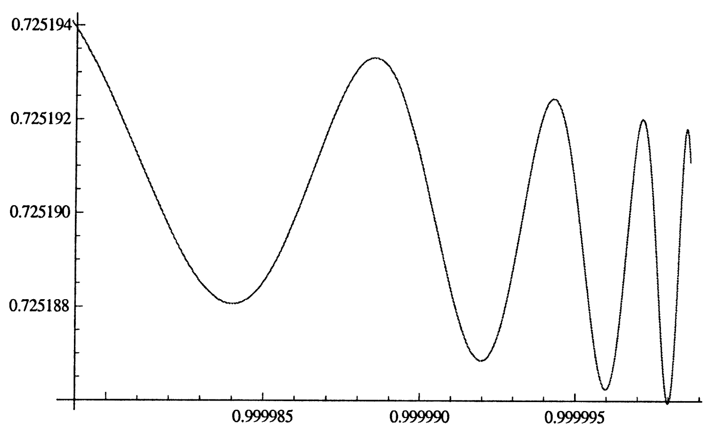

```
In[19]:= Plot[(1 - x)^ga f[x, 25], {x, t[13], t[17]}]
```

Out[19]=



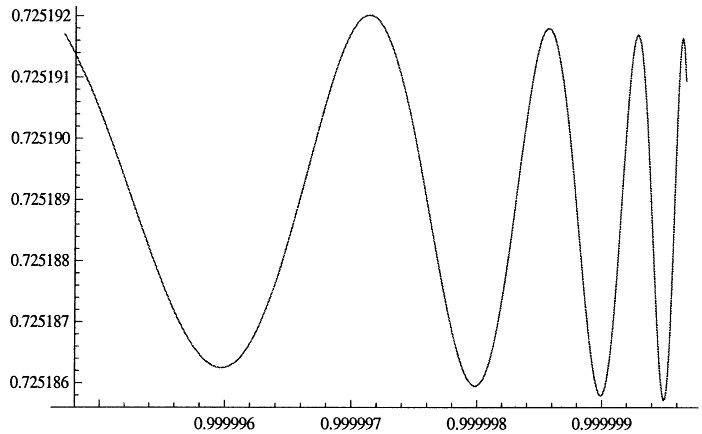

```
In[20]:= Plot[(1 - x)^ga f[x, 25], {x, t[15], t[19]}]
```

Out[20]=



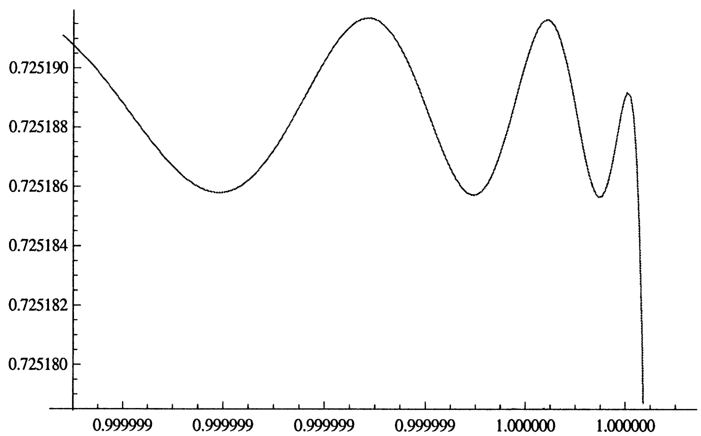
In[21]:=
```
Plot[(1 - x) ^ga f[x, 25], {x, t[17], t[21]}]
```

Out[21]=



In[22]:=
```
Plot[(1 - x) ^ga f[x, 25], {x, t[19], t[23]}]
```

Out[22]=

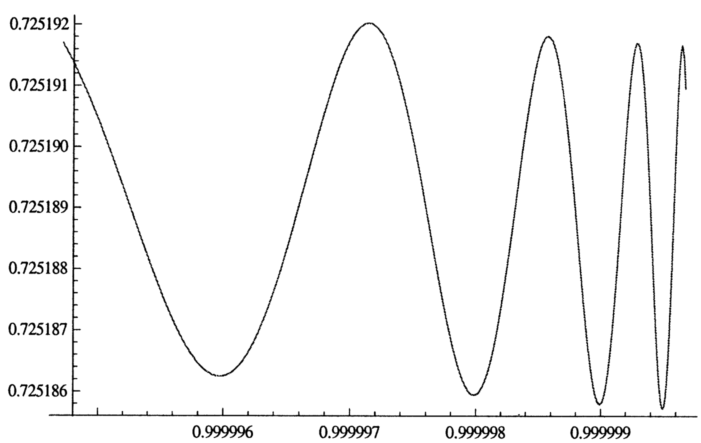


In[24]:=
```
N[{t[21] ^ (2 ^ 26), t[23] ^ (2 ^ 26)}]
```

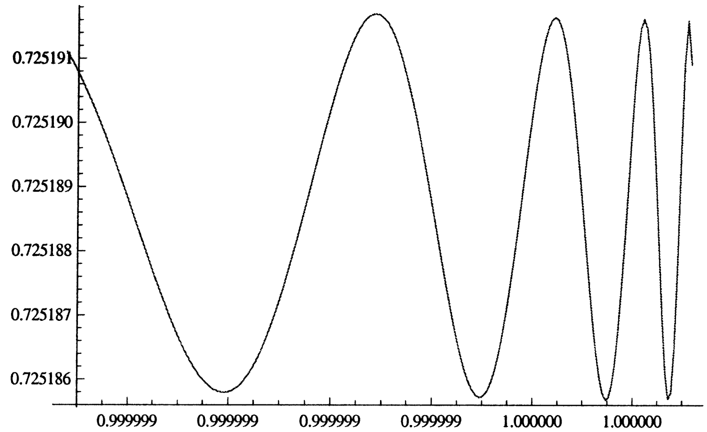
Out[24]= $\{2.32831 \times 10^{-10},\ 0.00390625\}$

In[25]:=
```
Plot[(1 - x) ^ga f[x, 30], {x, t[17], t[21]}]
```

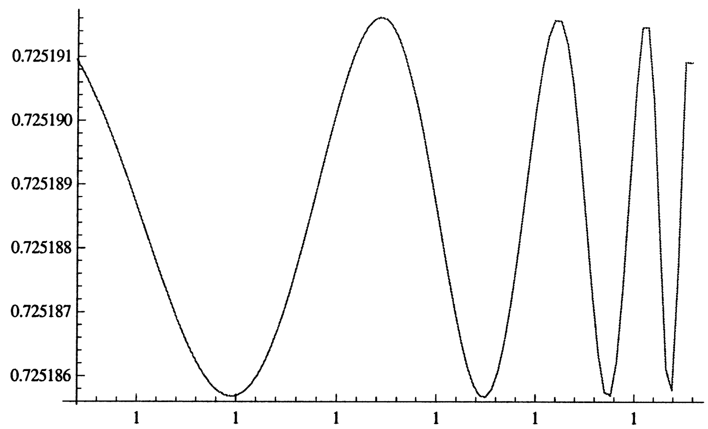
Out[25]=



In[26]:= `Plot[(1 - x) ^ga f[x, 30], {x, t[19], t[23]}]`

Out[26]=


In[27]:= `Plot[(1 - x) ^ga f[x, 30], {x, t[21], t[25]}]`

Out[27]=


In[28]:= `N[t[25] ^ (2 ^ 30)]`

Out[28]= 2.32831×10^{-10}

In[29]:= `N[{1 - t[25], 1 - t[21]}]`

Out[29]= $\{2.06574 \times 10^{-8},\ 3.30518 \times 10^{-7}\}$

In[30]:=
```
g[x_, n_] :=
  Product[Abs[(1 + x^(2^k) + x^(2^(k+1)))], {k, 0, n}]
```

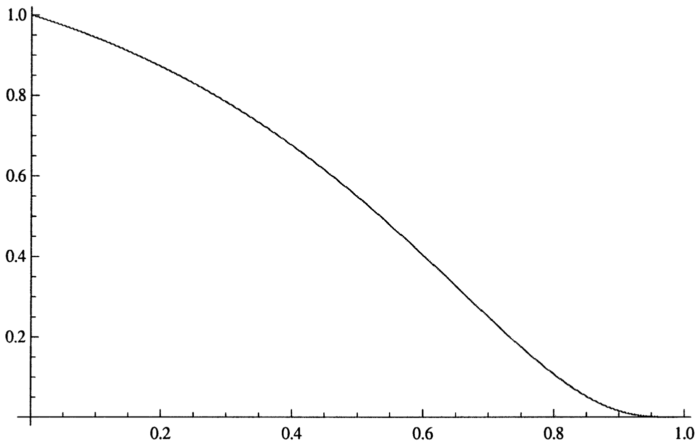
In[33]:= `om = (-1 + Sqrt[-3]) / 2;`

In[36]:= `{om, om^3, N[om^3]}`

Out[36]= $\left\{\frac{1}{2}\left(-1 + i\sqrt{3}\right),\ \frac{1}{8}\left(-1 + i\sqrt{3}\right)^3,\ 1. + 1.11022 \times 10^{-16}\, i\right\}$

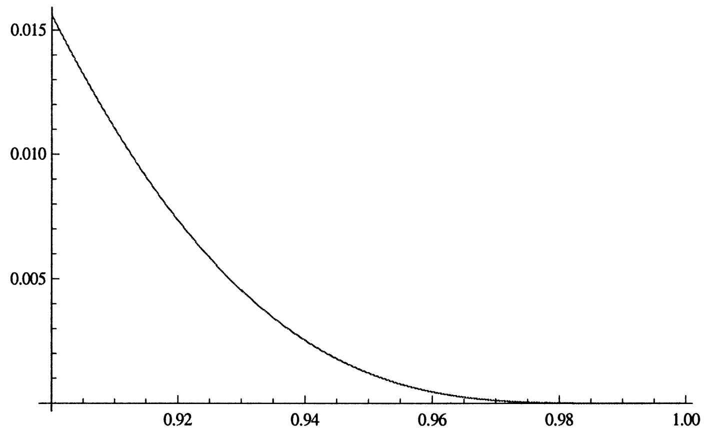
In[40]:= `Plot[g[om x, 20], {x, 0, .99}]`

Out[40]=



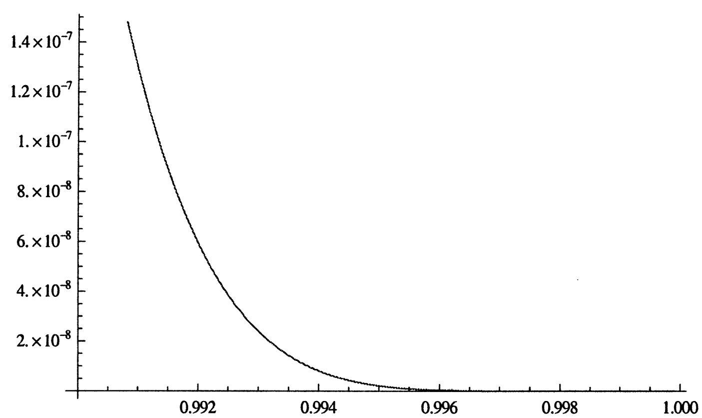
In[41]:= `Plot[g[om x, 20], {x, .9, .999}]`

Out[41]=



In[42]:= `Plot[g[om x, 20], {x, .99, .9999}]`

Out[42]=

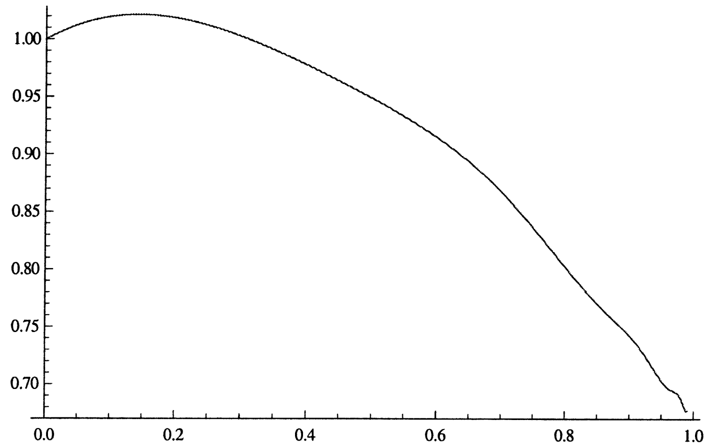


In[43]:= `eta = E^(2 Pi I / 5)`

Out[43]= $e^{\frac{2 i \pi}{5}}$

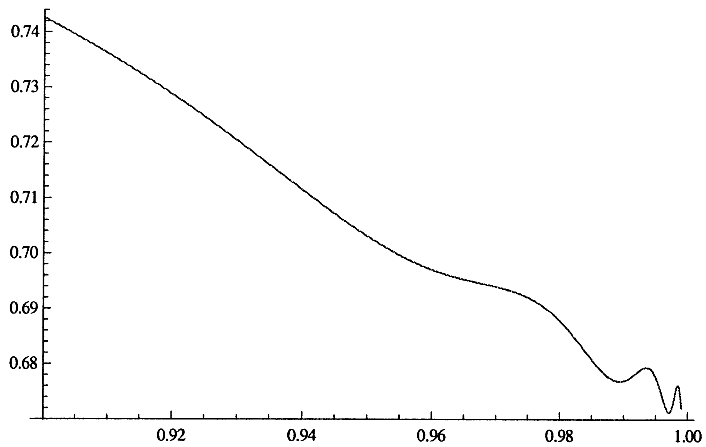
In[44]:= **Plot[g[eta x, 20], {x, 0, .99}]**

Out[44]=



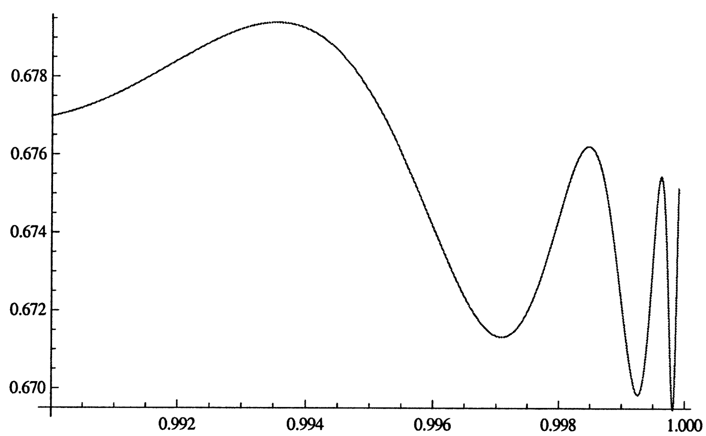
In[45]:= **Plot[g[eta x, 20], {x, .9, .999}]**

Out[45]=



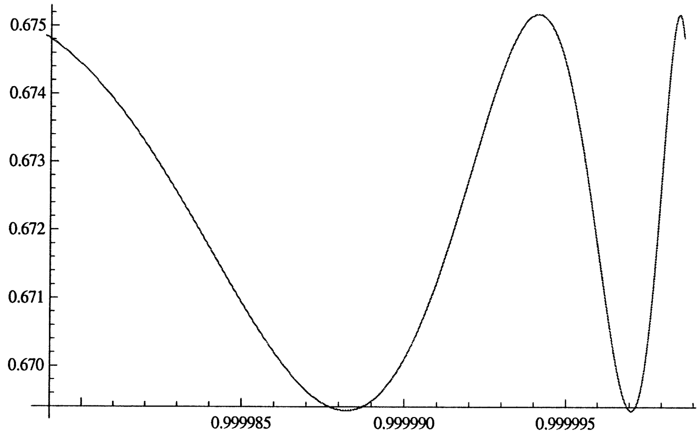
In[46]:= **Plot[g[eta x, 20], {x, .99, .9999}]**

Out[46]=



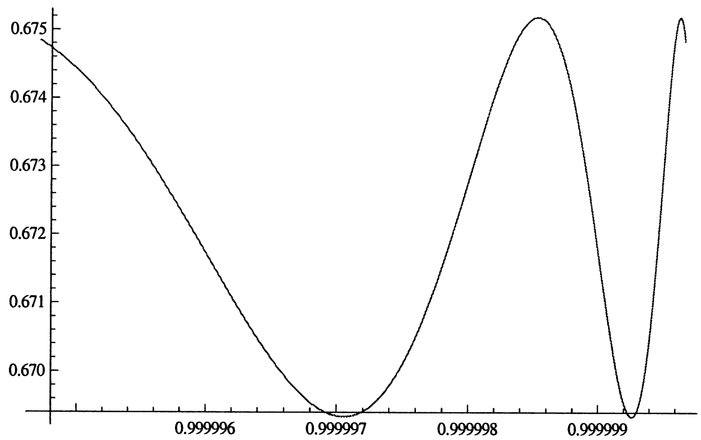
In[47]:= `Plot[g[eta x, 30], {x, t[15], t[19]}]`

Out[47]=



In[49]:= `Plot[g[eta x, 30], {x, t[17], t[21]}]`

Out[49]=

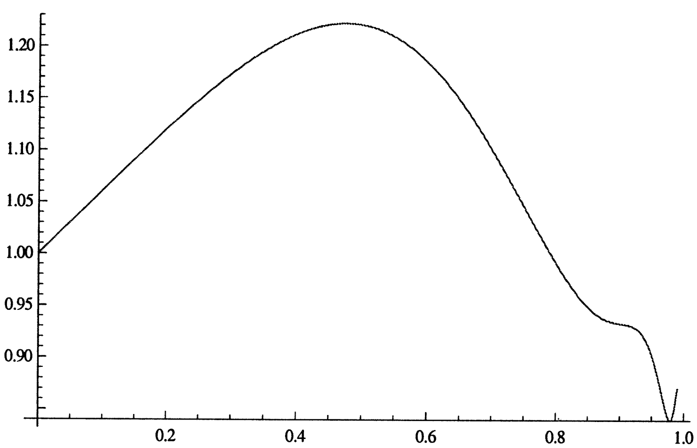


In[50]:= `nu = E^(2 Pi I / 7)`

Out[50]= $e^{\frac{2 i \pi}{7}}$

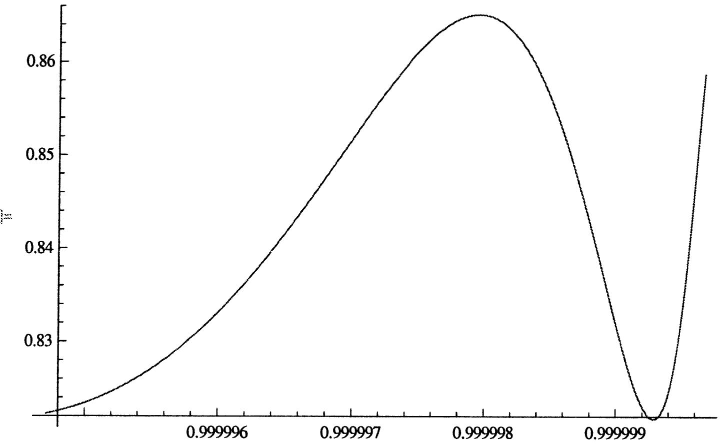
In[51]:= `Plot[g[nu x, 20], {x, 0, .99}]`

Out[51]=



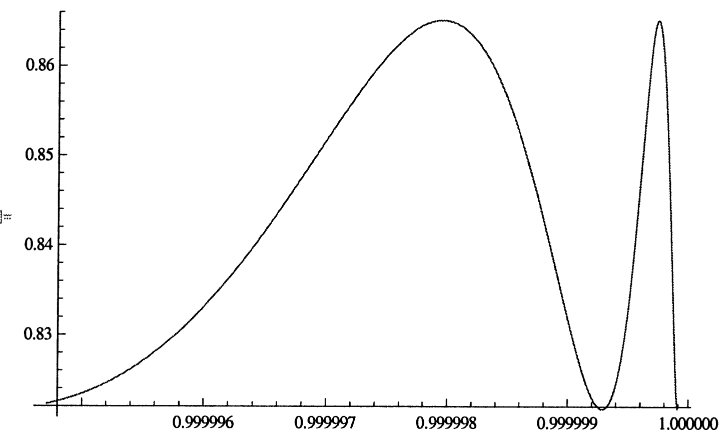
In[57]:= `Plot[g[nu x, 30], {x, t[17], t[21]}]`

Out[57]=



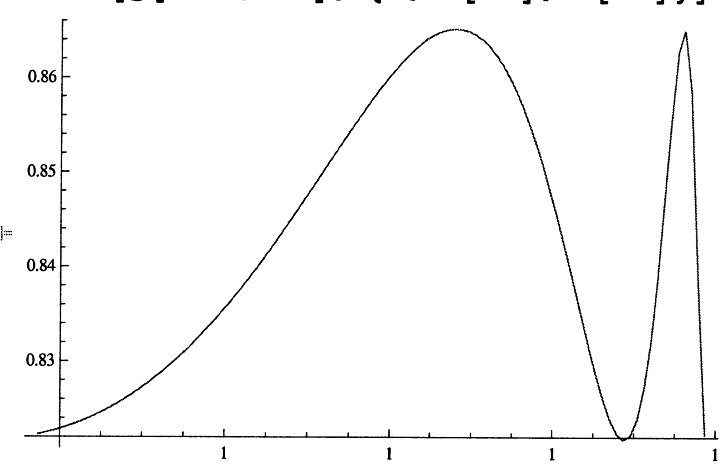
In[58]:= `Plot[g[nu x, 30], {x, t[17], t[23]}]`

Out[58]=



In[60]:= `Plot[g[nu x, 35], {x, t[23], t[29]}]`

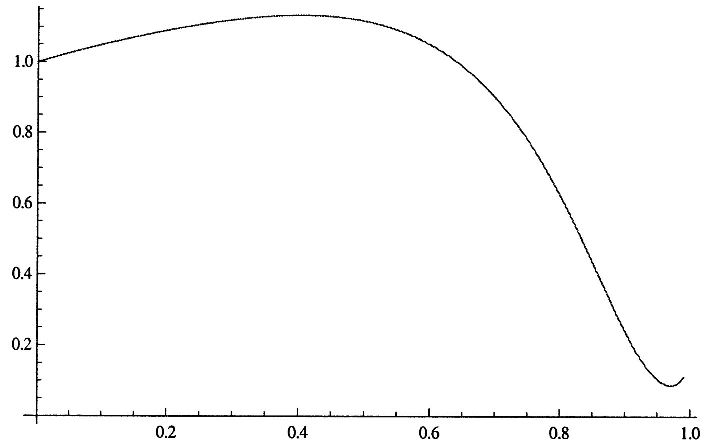
Out[60]=



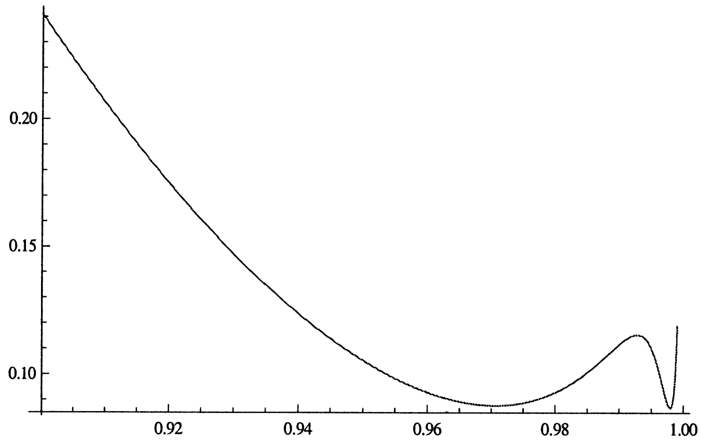
In[61]:= `xi = E ^ I`

Out[61]= e^{i}

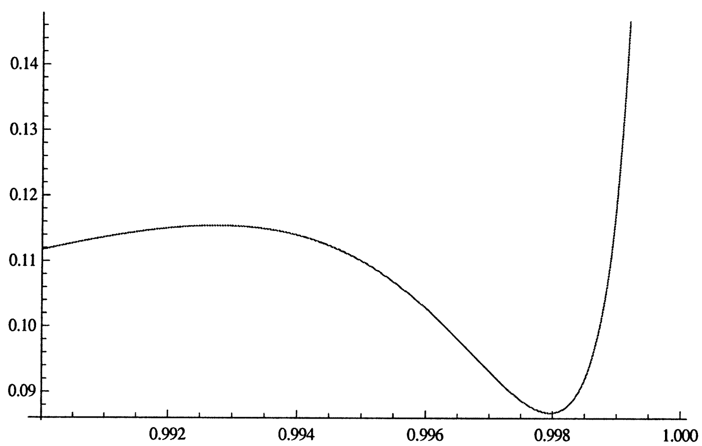
In[64]:= **Plot[g[xi x, 20], {x, 0, .99}]**

Out[64]= 

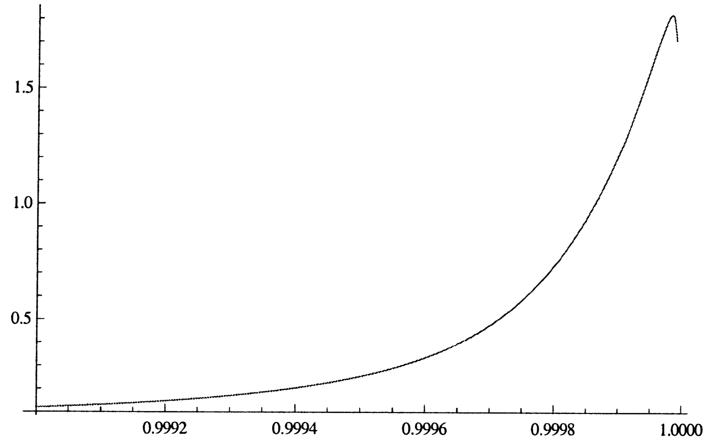
In[65]:= **Plot[g[xi x, 25], {x, .9, .999}]**

Out[65]= 

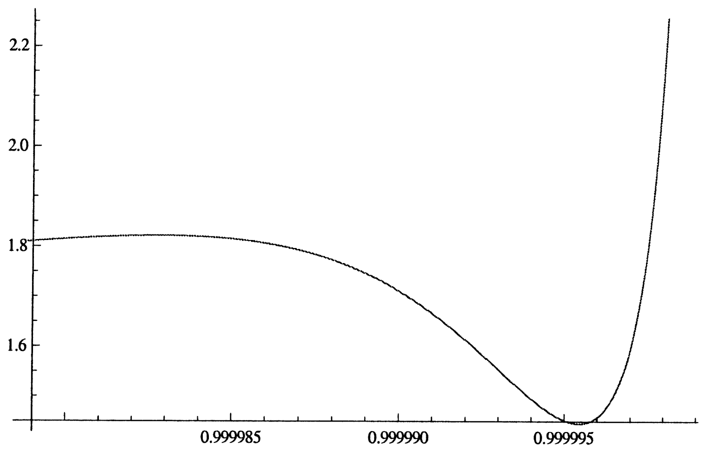
In[66]:= **Plot[g[xi x, 25], {x, .99, .9999}]**

Out[66]= 

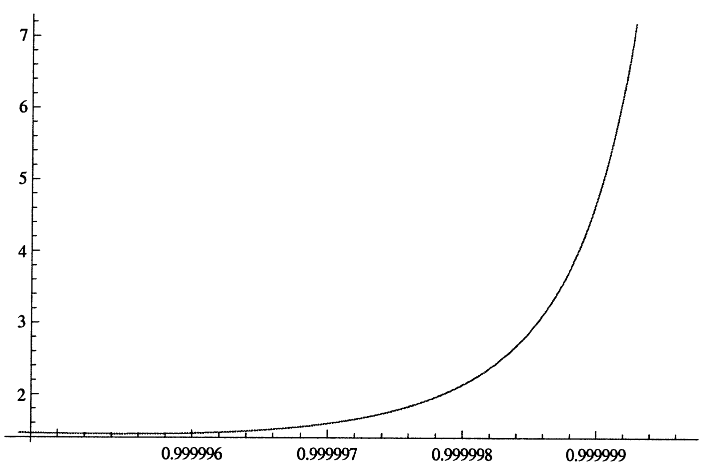
In[67]:= Plot[g[xi x, 25], {x, .999, .99999}]

Out[67]=


In[68]:= Plot[g[xi x, 25], {x, t[15], t[19]}]

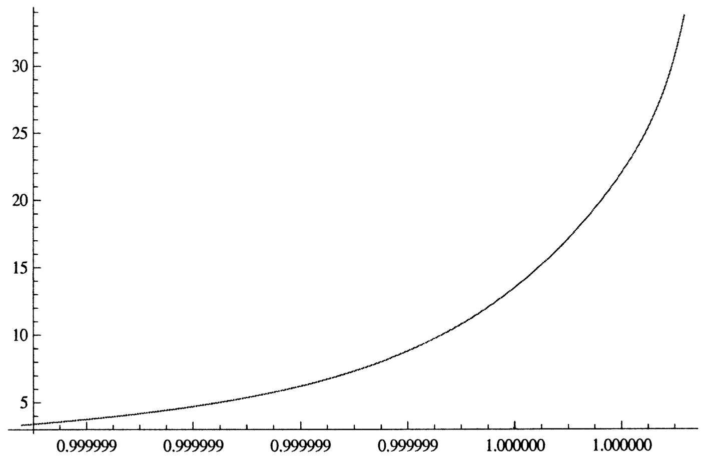
Out[68]=


In[69]:= Plot[g[xi x, 25], {x, t[17], t[21]}]

Out[69]=


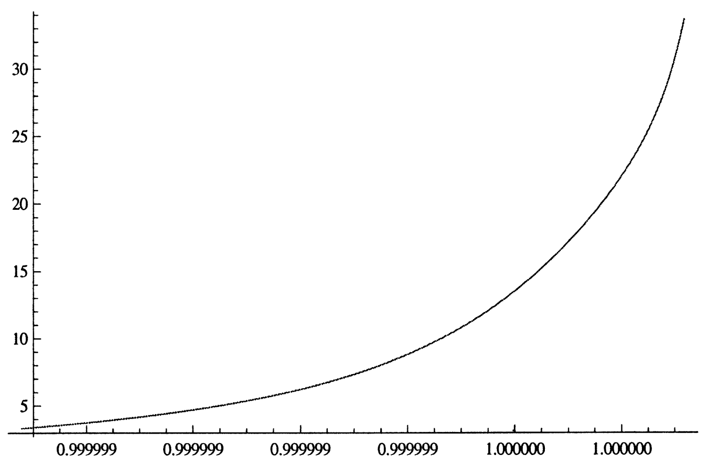
In[70]:= `Plot[g[xi x, 25], {x, t[19], t[23]}]`

Out[70]=



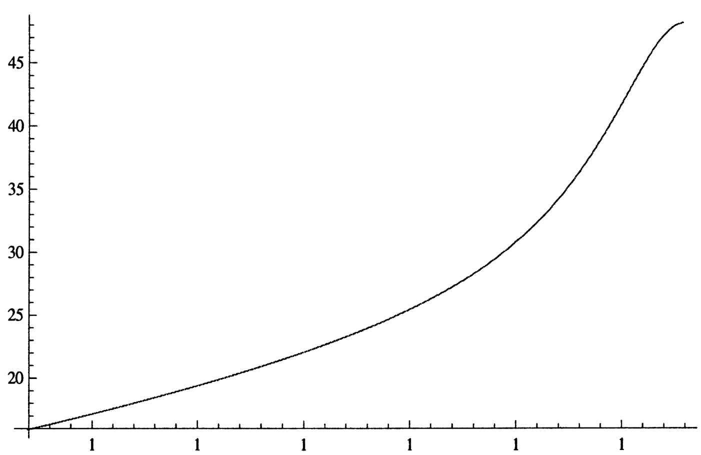
In[71]:= `Plot[g[xi x, 30], {x, t[19], t[23]}]`

Out[71]=



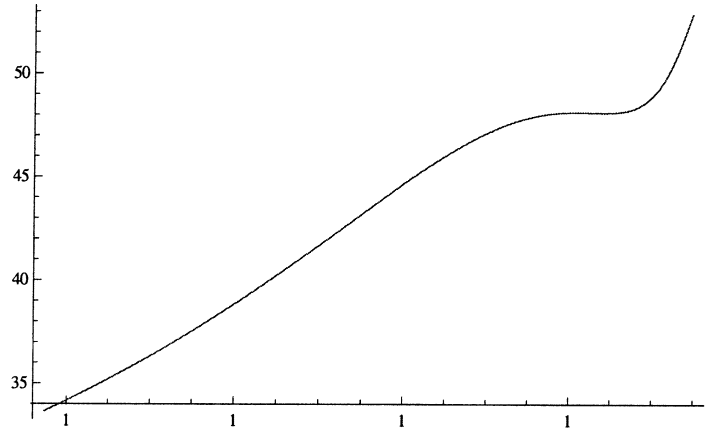
In[72]:= `Plot[g[xi x, 30], {x, t[21], t[25]}]`

Out[72]=



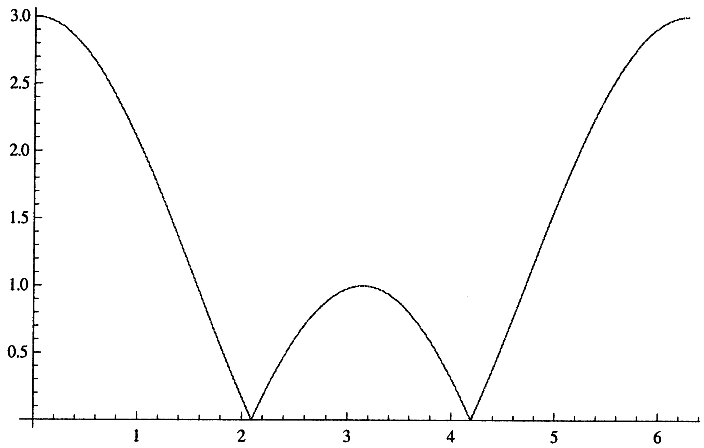
In[73]:= **Plot[g[xi x, 30], {x, t[23], t[27]}]**

Out[73]=



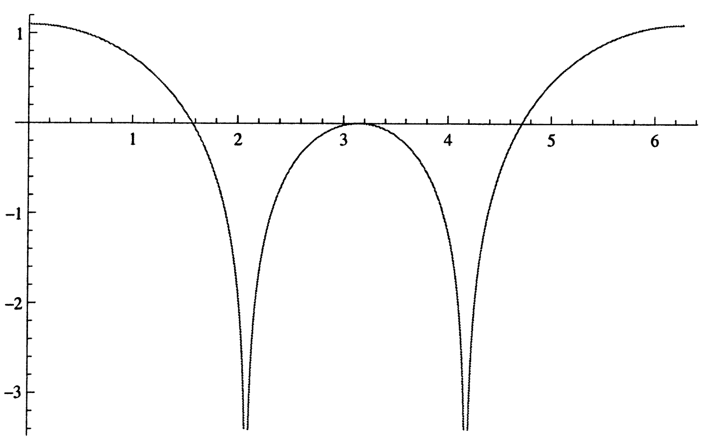
In[74]:= **Plot[Abs[1 + 2 Cos[t]], {t, 0, 2 Pi}]**

Out[74]=



In[75]:= **Plot[Log[Abs[1 + 2 Cos[t]]], {t, 0, 2 Pi}]**

Out[75]=



In[76]:= **NIntegrate[Log[Abs[1 + 2 Cos[t]]], {t, 0, 2 Pi}]**

NIntegrate::ncvb : NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in t near {t} = {4.18879024234974820131745015343633265110512287066057979245670139790}. NIntegrate obtained $1.09981\times10^{-15}+9.54287\times10^{-16}\,i$ and 7.659232274947312`*^-12 for the integral and error estimates. >>

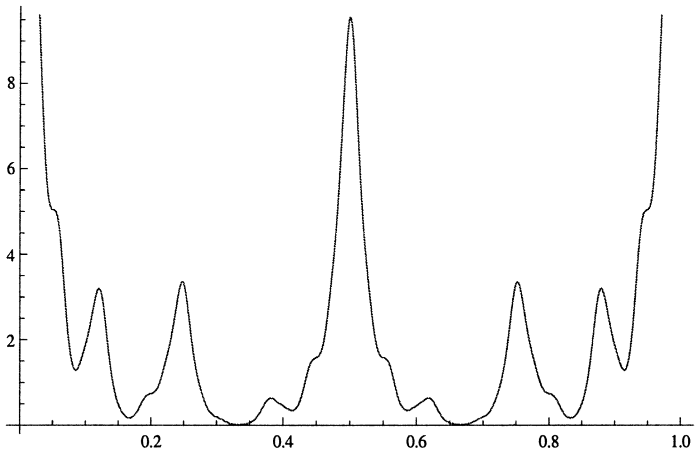
Out[76]= $1.09981\times10^{-15} + 9.54287\times10^{-16}\,i$

In[80]:= **N[.9^(2^10)]**

Out[80]= 1.39421×10^{-47}

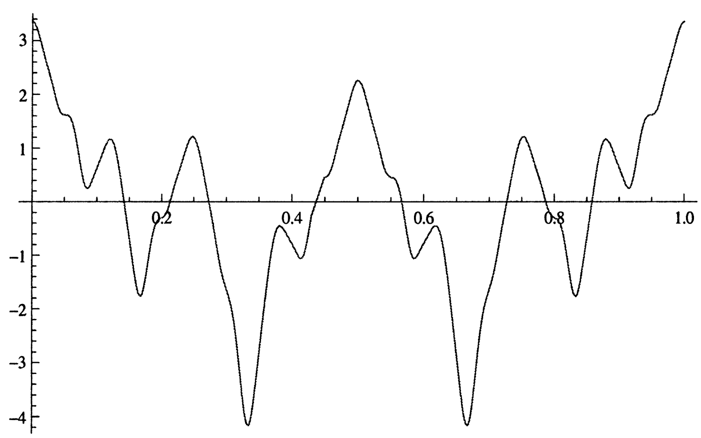
In[81]:= **Plot[g[.9 E^(2 Pi I t), 10], {t, 0, 1}]**

Out[81]=



In[82]:= **Plot[Log[g[.9 E^(2 Pi I t), 10]], {t, 0, 1}]**

Out[82]=

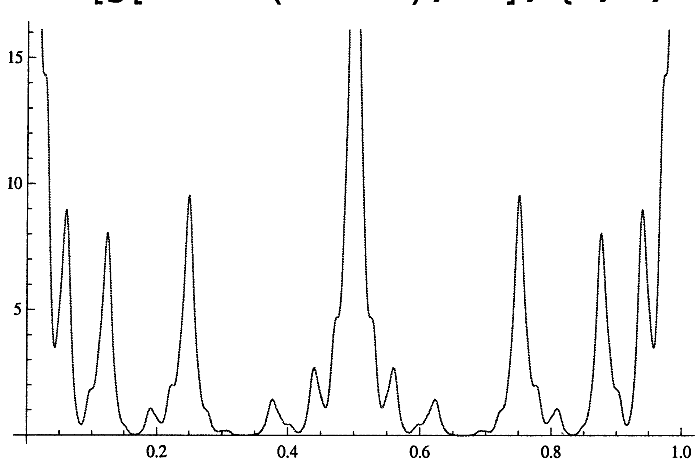


In[83]:= **N[.95^(2^10)]**

Out[83]= 1.54515×10^{-23}

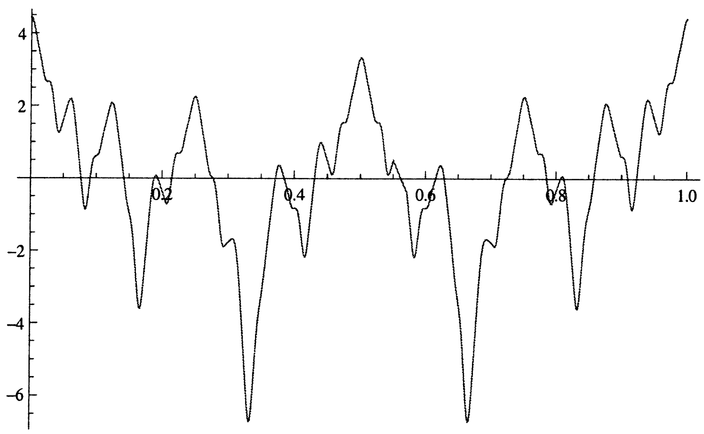
In[84]:= **Plot[g[.95 E^(2 Pi I t), 10], {t, 0, 1}]**

Out[84]=



In[85]:= **Plot[Log[g[.95 E ^ (2 Pi I t), 10]], {t, 0, 1}]**

Out[85]=

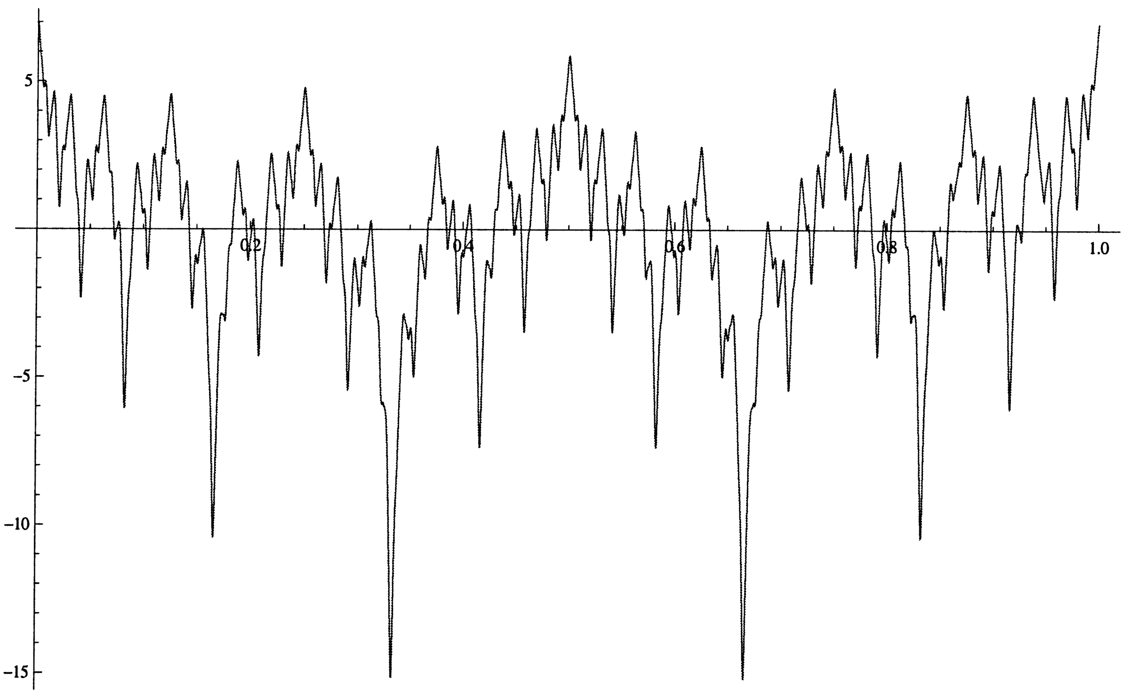


In[87]:= **N[.99 ^ (2 ^ 12)]**

Out[87]= 1.3236×10^{-18}

In[88]:= **Plot[Log[g[.99 E ^ (2 Pi I t), 12]], {t, 0, 1}]**

Out[88]=



In[90]:= **g[.99, 12]**

Out[90]= 1074.68

In[91]:= **g[.99 om, 12]**

Out[91]= 2.5579×10^{-7}

In[95]:= **{f[.9, 10], f[.9, 20]}**

Out[95]= {28.49, 28.49}

In[97]:= **Sum[s[n] * .9 ^ (n - 1), {n, 1, 100}]**

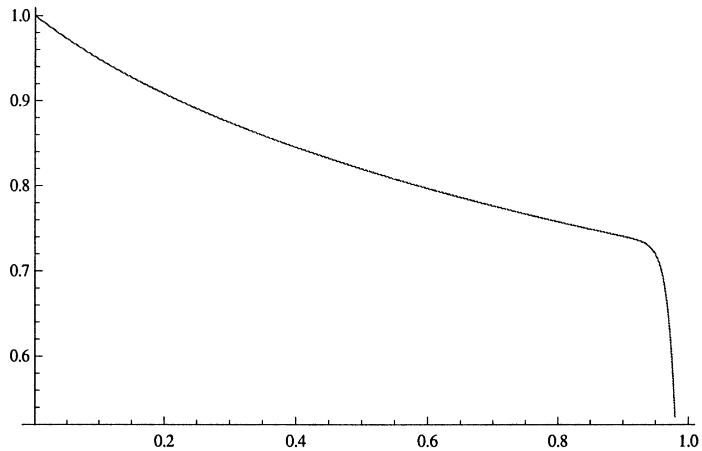
Out[97]= 28.4865

In[98]:= **.9 ^ 99**

Out[98]= 0.0000295127

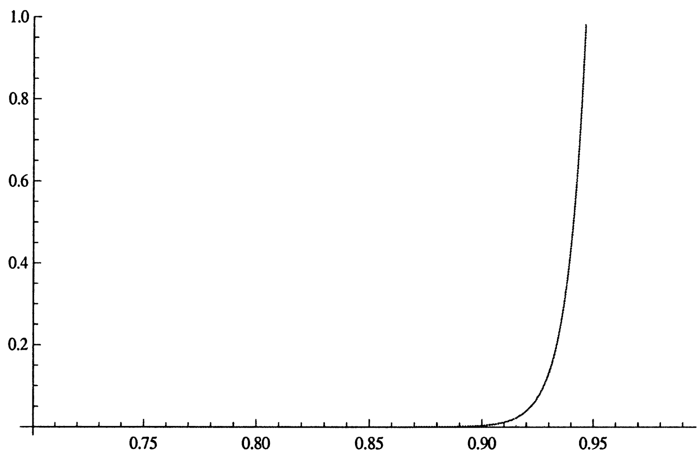
In[100]:= **Plot[(1 - x) ^ga Sum[s[n] * x ^ (n - 1), {n, 1, 100}], {x, 0, .99}]**

Out[100]=



In[105]:= **Plot[f[x, 25] - Sum[s[n] * x ^ (n - 1), {n, 1, 100}], {x, .7, .99}]**

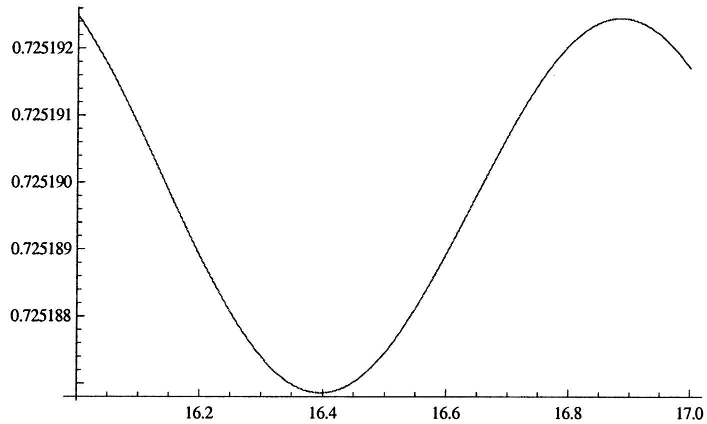
Out[105]=



In[114]:= h[x_, n_] := Product[1 + t[x - j] + t[x - j - 1], {j, 0, n}]

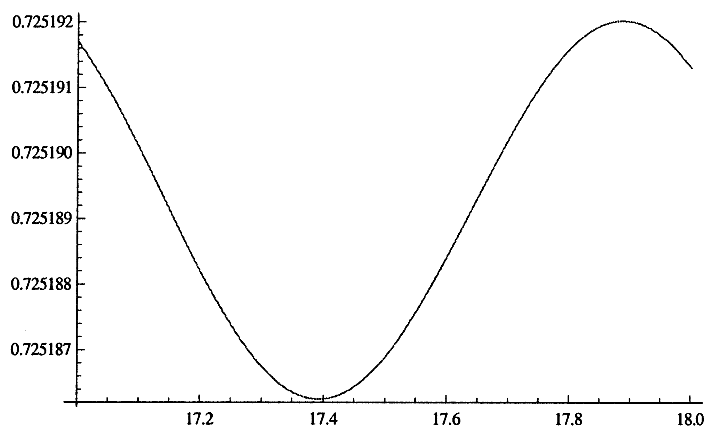
In[116]:= Plot[(1 - t[x]) ^ (ga) h[x, 25], {x, 16, 17}]

Out[116]=



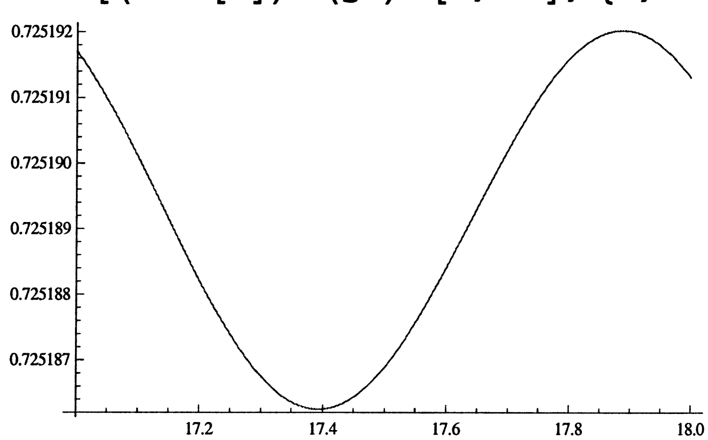
In[117]:= Plot[(1 - t[x]) ^ (ga) h[x, 25], {x, 17, 18}]

Out[117]=



In[118]:= Plot[(1 - t[x]) ^ (ga) h[x, 30], {x, 17, 18}]

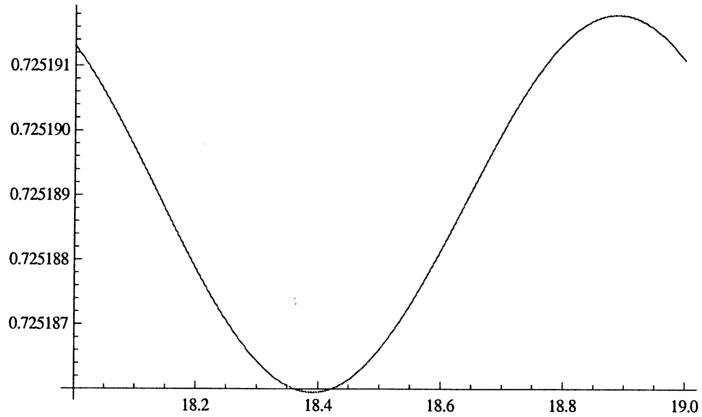
Out[118]=



In[119]:=

```
Plot[(1 - t[x]) ^ (ga) h[x, 30], {x, 18, 19}]
```

Out[119]=



In[120]:=

```
Plot[(1 - t[x]) ^ (ga) h[x, 30] -
  (1 - t[x - 1]) ^ (ga) h[x - 1, 30], {x, 18, 19}]
```

Out[120]=