Instructor: Prof. Bruce Reznick, 327 Altgeld Hall, 333–4284, reznickmath.uiuc.edu. My phone has voice mail and I frequently check and reply to my email, including weekends. Office hours are by appointment. I take them seriously, and they can usually be arranged within 24 hours. You are also encouraged to ask me questions immediately before, during and after class. I’m terrible with names; don’t take it personally. This course has a webpage: http://www.math.uiuc.edu/~reznick/math496F12.html.

There will be a “class diary” summarizing the class day by day, providing links to handouts, and containing my responses to all questions sent to me by email. If you email me a course question, I will post your anonymized question and my reply, for the benefit of the entire class. I also expect to have an ongoing page of useful mathematical links, to which you are invited to contribute.

The Blurb: Few prospects are as daunting for a serious math major as that of doing research. In fact, mathematical research is a natural extension of homework in mathematics courses, except that there is no back of the book to look for the answer. This course is designed to help students develop their skills in mathematical creativity and problem-solving and (These skills are useful in all advanced mathematics classes as well.) The only formal prerequisite is Math 347, or the ability to convince the instructor that you can write proofs correctly. However, this is an honors course in terms of the approach to the subject.

Some of the ways in which research differs from homework are these: the problems are harder, you may not know whether you (or anyone else) can solve them, you do not always know what you need to know in order to solve them, they are usually motivated by a larger set of questions and, ultimately, by the researcher’s own curiosity. Research is also often a matter of synthesizing several seemingly different results into a cohesive whole.

The Course: The best way for a student to become successful in mathematical research is to take as many challenging and meaningful mathematics courses as possible, in many different areas. This course, on the other hand, concentrates on building an infrastructure for research. In the first part of the semester, we will consider problem-solving, question-asking, answer-analyzing and knowledge-finding, and you will choose a project and start working on it. Mathematical creativity is a subset of human creativity, and much is known about how to become more creative. In the second part of the semester, students will present their own research projects and listen to and critique the work of the others. Research projects can be individual or group, at your discretion. These projects can be used as a basis for the Senior Paper or as a submission to the Greenwood-Trjitzinsky Prize. The instructor’s own Ph.D. thesis began as an undergraduate project, but he cannot guarantee this outcome!

There will be students at varying levels of mathematical knowledge and sophistication in this class. Don’t be scared off by thinking that you don’t know enough mathematics — nobody ever knows enough mathematics. (Your professors are continually learning new mathematics.) Class members may work on any approved mathematical research topic; a range of topics will also be provided for those who request it. It’s OK to work in a related area (e.g. computer science, economics, physics, statistics), as long as the research itself has a
serious mathematical component. Collaboration is both acceptable and strongly encouraged, and is good practice for the “real” research you’ll do later.

Since this class is a seminar, you will be expected to participate actively and attend nearly every class meeting. The standard “unit of talk” for a student is a quarter period, including time for questions, except for the final presentation, which is half a period. I will postpone any presentation I have planned if you get excited about something mathematically and want to share it with the group. During the first few weeks, you’ll be asked to make at least one presentation on a mathematical topic you find interesting. By Spring Break, you will have selected your research topic, and you’ll be asked to talk about it in the middle of the semester. Finally, you’ll be asked to talk about your progress by the end of the semester.

This is an experimental course, and the above description represents the instructor’s expectation, without accounting for student input into the organization. Changes suggested by the previous groups of students will be incorporated: (i) You should (for your own sake) keep a “discovery notebook” to record your ideas throughout the semester – this will not be collected; (ii) You should turn in a serious draft of your final report at least a week before the semester ends, so that I can make seriously detailed suggestions for your final report. I encourage you to be assertive in giving me feedback on the way things are going: my ambition is that this be the most successful course in the history of undergraduate mathematics, and I’ll settle for 90% of that. If ICES forms can be believed, previous students have found this course valuable. Become an active participant – let it get under your skin and visit your dreams. These are serious steps towards becoming a mathematician.

The Texts: I have written an article based on the first few weeks of this course which appeared in Math Horizons a few years ago: it can be found at http://www.math.uiuc.edu/~reznick/mhori.pdf

Two texts are recommended: “Mathematics and plausible reasoning: induction and analogy in mathematics” by George Pólya and “Proofs from the Book” by Martin Aigler and Gunter Ziegler. The first book is part of a classic series of books on problem solving by the man who coined the word “heuristics”. The second is a collection of short, accessible and beautiful proofs on a range of mathematical topics. Think of it as the Louvre. These books will provide many points of embarkation for your projects. Two other books which you might enjoy are “The man who loved only numbers” by Paul Hoffman, a biography of Paul Erdős, and “A mathematician’s apology” by G. H. Hardy, which is a classic and thoughtful essay on mathematics. There will be many handouts; I enjoy finding articles which amplify the presentations you have made in class.

Homework, Exam (what exam?) and Grading Policy: I will attempt a range of assignments in the first half of the semester. I expect you to concentrate your energies on your projects for the second half of the semester. I usually make the final project due at the time of the final exam and give a party at my house at a time which people can attend.

For a course such as this, I have no objection to giving very high grades overall, provided they reflect your effort and commitment to the class. Let me repeat: I will take your background into account when evaluating your work, and the more you put in to this course, the more you will take away.