(ungraded) Strayer – (Chapter Four) Problems 1ac, 10ac, 12a
Wolfram|Alpha problem: Pick two random integers, $k$ and $\ell$, between 100 and 1000, and ask for $m = \text{Prime}[k]$ and $n = \text{Prime}[\ell]$ Now ask for \( \left(\frac{m}{n}\right) \) and \( \left(\frac{n}{m}\right) \), either by asking if these are quadratic residues, or by using the formulation “JacobiSymbol[m, n].”

1. Strayer – Chapter Three, Problem 51.
2. Strayer – Chapter Four, Problems 1bd
3. Strayer – Chapter Four, Problem 10bd.
4. (E) This problem uses your number $N$. Determine the value of the Legendre symbol \( \left(\frac{N}{11}\right) \). You should be able to do this from the definition, without using quadratic reciprocity.
5a. (E) Solve the congruence $x^2 \equiv 1 \mod 253$. It helps to observe that $253 = 11 \cdot 23$ and to think about the Chinese Remainder Theorem.
5b. Solve the congruence $x^2 \equiv 1 \mod p(2p + 1)$ when $p$ and $2p + 1$ are both odd primes. Give your answer as $x \equiv 1, f_2(p), f_3(p), f_4(p) \mod 253$ for polynomials $f_j(p)$. It helps to think about your answer in part a!
6. (E) True or false: (I want either a short proof or a numerical counterexample. If a counterexample exists, then there is one in which the numbers involved are small.)
6a. There are exactly $12^3 - 12^2$ integers $a$ less than $12^3$ for which $\gcd(a, 12^3) = 1$.
6b. If $n > 6$ is an even perfect number, and $m = 3n$, then $\sigma(m) = 3m$.
7a. (E) Suppose $n$ and $m$ are integers and
$$\frac{\sigma(n)}{n} = \frac{m}{4}, \quad \gcd(m, 4) = 1.$$ 
Prove that $4 \mid n$.
7b. Find all integers $n$ for which
$$\frac{\sigma(n)}{n} = \frac{7}{4}.$$ 
7c. (Extra credit) Find with proof all integers $k$ and primes $p$ with the property that
$$\frac{\sigma(2^kp)}{2^kp} = \frac{9}{4}.$$ 
You will not receive any credit for a simple list of the integers; I want a proof that these are the only possible solutions.