Math 453  Homework 1  Due Friday, Sept. 6, 2019

The “ungraded” problems have their answers in the back. You are encouraged to work them and solutions will be provided, but they are, not graded and you don’t have to hand them in. They are occasionally the basis for exam questions.

The book numbers all problems in a chapter sequentially. The problems taken from the book on this assignment are all from Chapter 1. It may happen that part of a question is answered in the back of the book. You will not receive full credit unless you write down a full explanation. The symbol (E) means that at least part of this problem appeared on an old exam, up to possible numerical alterations.

It is important to write your proofs carefully and clearly. It is always a good idea to look at numerical versions of a proposed result before trying to prove it. Remember that one example does not show that a proposed theorem is true, but one counterexample does show that it is not true (but nothing more.)

(ungraded) Strayer – Problems 3e, 20, 21c, 32e, 36ab, 54c,

Wolfram|Alpha problem (ungraded): Find the prime factorization of you Social Security number, or your 10 digit phone number. (Change some digits for privacy.) What is the gcd of these two numbers?

1. Strayer – Problem 10. (Note that there is a hint to (a) in the back of the book.) Numerical examples may also be helpful; there are several correct methods.
3. This problem involves your integer \( N \), given to you on the Course Organization sheet. Show that \( \gcd(N, N + 3) = 1 \) and find two integers \( m \) and \( n \) (which depend on \( N \)) so that \( mN + n(N + 3) = 1 \).
4. (E) Compute \( g = \gcd(21, 35) \) and \( M = \text{lcm}(21, 35) \) by any correct method, and find two integers \( m \) and \( n \) so that \( g = 21m + 35n \).
5. (E) True or false: (I want either a short proof or a numerical counterexample. If a counterexample exists, then there is one in which the numbers involved are small.)
   (i) If \( a \) and \( b \) are integers, and \( \gcd(a, b) > 1 \), then \( \gcd(a + 1, b + 1) = 1 \).
   (ii) If \( \gcd(a, b) = 1 \) and \( ab \mid m \), then \( \gcd(a, \frac{m}{ab}) = 1 \). (Hint: it is helpful to write \( m = abk \), with \( k \in \mathbb{Z} \).)
6. (E) Suppose \( p \neq 2, 3 \) is prime and \( 4p + 1 \) is also prime. Prove that \( 8p + 1 \) cannot be prime. (Hint: there exists an integer \( m \) such that either \( p = 3m + 1 \) or \( p = 3m + 2 \).)
7. Prove that \( n \) does not divide \( (n - 1)! \) if and only if \( n \) is prime or \( n = 4 \).
   A correct solution will consist of two proofs:
   (i) If \( n \) is prime or \( n = 4 \) then \( n \nmid (n - 1)! \).
   (ii) If \( n > 4 \) is composite, then \( n \mid (n - 1)! \). Hint: you should treat the case where \( n \) is a square with some care.