You know the rules by now. The problems in this homework are from Chapters Five and Six. It is important to write your proofs carefully and clearly. It is important to cite results in the book if you write down an answer without doing much computation.

(ungraded) Strayer – Ch.5 – 24a, 30c, 33a; Ch.6 – 2ab

1. Strayer – Ch.5 – 19.
2. Strayer – Ch.5 – 24b, 33b
3. Strayer – Ch.6 – 4. (I’d choose a different caterer.)
4. (E) Determine the number of solutions of the equation
   \[ x^3 \equiv x^2 \pmod{50}. \]
   Hint: \( x^3 - x^2 = x^2(x-1) \). You should also factor 50 as well. (This doesn’t specifically use the work of chapter 5, but is good review.)

5. (E) You are told that 3 is a primitive root modulo 353. Given this information, solve the equation \( x^{12} \equiv 81 \pmod{353} \). Leave your answer in the form \( x \equiv 3^{k_1} \pmod{353} \) for specific integers \( k_1, \ldots \) (as many as you need.)

6. (E) Suppose \( p \) and \( q \) are odd primes and \( q \mid 5^p - 1 \). Prove that \( q \equiv 1 \pmod{p} \). Hint 1: think about the question: “Can \( ord_q 5 = 1 \)?” Hint 2: This result is false with “5” replaced by “7” – consider \( p = q = 3 \) and note that \( 3 \mid 7^3 - 1 \).

7. (E) Find all positive integers \((x, y)\) with the property that \( 35^2 + y^2 = z^2 \). There is no assumption about \( \gcd(y, z) \). (Note: 35 = 5 \cdot 7.)

8. Extra credit problem: Suppose \( p \) is an odd prime, and \( a \) is an integer so that \( \left( \frac{a}{p} \right) = -1 \). Prove that, if the Diophantine equation
   \[ x^2 + py^2 = az^2. \]
   has positive integer solutions \( x, y, z \), then \( p \) divides both \( x \) and \( z \).