1. Find all solutions in positive integers to:

\[ 4x + 6y = 63, \quad 4x + 6y = 62 \]

\[ \gcd(4, 6) = 2, \quad 2 \mid 63, \text{ so no solutions here!} \]

\[ 4x + 6y = 62 \iff 2x + 3y = 31 \quad \implies 2(-1) + 3(1) = 1 \]

So we can take \( x_0 = -31, y_0 = 31 \) and \( 2x_0 + 3y_0 = 31 \)

\[ x = x_0 + 3t = -31 + 3t \quad \text{Positive integers: } 3t - 31 \geq 0 \rightarrow t \geq 10.33 \]
\[ y = y_0 - 2t = 31 - 2t \quad \rightarrow 15.5 \geq t \]

\[
\begin{array}{c|c|c}
\times & 2 & 9 \\
12 & 5 & 7 \\
13 & 8 & 5 \\
15 & 11 & 3 \\
\end{array}
\]

\[ 4 \cdot 2 + 6 \cdot 9 = 8 + 54 = 62 \]

Note: maybe you picked a different \( (x_0, y_0) \). That's fine. But the positive solutions should be the same!

2. Find a primitive Pythagorean triple \((a, b, c)\)

so that one of \([a, b, c]\) is \(105 = 3 \cdot 5 \cdot 7\)

\[ a = m^2 - n^2 \quad b = 2mn \quad c = m^2 + n^2 \]

Let's try \( m^2 - n^2 = 105 = (m+n)(m-n) \)

There are several choices:

1. \( m+n = 35 \) \quad \( m-n = 3 \) \quad \( m = 19 \) \quad \( n = 16 \)
2. \( m+n = 21 \) \quad \( m-n = 5 \) \quad \( m = 13 \) \quad \( n = 8 \)
3. \( m+n = 15 \) \quad \( m-n = 7 \) \quad \( m = 11 \) \quad \( n = 4 \)
4. \( m+n = 105 \) \quad \( m-n = 1 \) \quad \( m = 53 \) \quad \( n = 52 \)

So \( a = 105 \)

1. \( 105, 605, 617 \)
2. \( 105, 205, 233 \)
3. \( 105, 68, 137 \)
4. \( 105, 5512, 5513 \)

Note: \( m^2 - n^2 = 105 \)

\[ \implies m^2 + n^2 \equiv 0 \mod 3 \]

as we have seen. Two numbers:

\[ m = 0 \mod 3, \quad n = 0 \mod 3, \quad \text{so} \]

\[ 3 \mid m, 3 \mid n, \quad \text{so } 3 \mid m^2 + n^2 \]

as \( 9 \mid 105 \)

so no solutions with \( c = 105 \)