Math 448  Homework 5  Due Friday, October 1, 2010

This is the next-to-last homework assignment on material that will be on the first test.

(ungraded) §2.3 – 1, 3, 5, 7, 9, 11 WolframAlpha: Ask for \( \int_{-\infty}^{\infty} \frac{p(x)}{q(x)} \, dx \) for your choice of polynomials \( p, q \), where \( \deg p \leq \deg q + 2 \). Mathematica knows Cauchy’s Theorem!

1. (graded) §2.3 – Problems 2 and 4.
2. (graded) §2.3 – Problems 10 and 12.
3. and 4. (graded) §2.3 – Problems 14, 15 and 16. (These are closely linked, and 15b isn’t phrased well. The function \( f(x, y) \) has a strict local maximum at \((x_0, y_0)\) if there exists \( r > 0 \) so that \( f(x_0, y_0) > f(x, y) \) for all points \((x, y) \neq (x_0, y_0)\) whose distance from \((x_0, y_0)\) is less than \( r \). Please note that in #16, the condition that \( f \neq 0 \) is a necessary hypothesis: if \( f(z) = z \), then \(|f|\) has a strict local minimum at \( z = 0 \). This suggests that whatever method you might want to apply won’t work if \( f \) takes the value zero.)
5. (graded) (E) Evaluate the following integrals, where \( C \) denotes the circle \(|z| = 2\), taken in the usual counterclockwise direction;
   \[
   \frac{1}{2\pi i} \int_C \frac{\cos z}{z} \, dz; \quad \frac{1}{2\pi i} \int_C e^{43z}(z-1)^5 \, dz.
   \]
6. and 7. (graded) (E) Use Cauchy’s formula to evaluate
   \[
   \int_{|z|=1} \frac{dz}{(4z-1)(z-4)},
   \]
   being careful about factors of 4, \( 2\pi i \), etc. Then substitute \( z = e^{i\theta} \) into your answer to find specific real numbers \( a, b, c \) so that your answer implies that
   \[
   \int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = c.
   \]
8. (bonus) This problem isn’t hard if you look at it the right way.
   a. Suppose \( C \) is a simple, piecewise smooth (not necessarily closed) contour. Prove that \( \int_C z \, dz = 0 \) implies that \( \int_C z^3 \, dz = 0 \).
   b. Find a simple, piecewise smooth contour \( C \) with the property that \( \int_C z \, dz = 1 \) and \( \int_C z^3 \, dz = 0 \).
9. and 10. (bonus) Let \( D \) consist of the complex plane minus the negative real axis. Let \( f(z) = \text{Exp}(\frac{1}{4} \text{Log} \, z) \) on \( D \); specifically, if \( z = re^{it} \), where \(-\pi < t < \pi\), let
   \[
   f(re^{it}) = r^{1/4}e^{it/4}.
   \]
   (This defines a “branch” of \( z^{1/4} \).) Use the method of Theorem 3, p. 109, to construct an explicit anti-derivative for \( f \) on \( D \), using \( z_0 = 1 \) as your “basepoint”. Hint: the easiest integrations come from connecting 1 to \( z = re^{it} \) by the arc from 1 to \( e^{it} \), followed by the ray from \( e^{it} \) to \( re^{it} \).