Math 448  Homework 3  Due Friday, September 17, 2010

Same rules as last week; ask me if you have questions.

(ungraded) §1.6 – 1,3,5, §2.1 – 1a, 3, 9. Wolfram|Alpha: choose two random non-zero digits, $m$ and $n$ and compute the integral of $z^m$ on the $|z| = n$, taken counterclockwise of course. Check on Wolfram|Alpha, with $z = ne^{it}, 0 \leq t \leq 2\pi$.

1. (graded) §1.6 – Problem 2.
2. (graded) §1.6 – Problem 6.
3. (graded) §1.6 – Problem 12.
4. (graded) §2.1 – Problems 2, 4 and 6.
5. (graded) §2.1 – Problem 16.
6. (graded) (E) Let $C$ denote the half-circle from $z = 4$ to $z = -4$, taken in the usual counterclockwise fashion. Calculate each of the following integrals by any correct method:
   
   \[ \int_C z^2 \, dz, \quad \int_C |z|^2 \, dz, \quad \int_C \bar{z} \, dz. \]

7. (graded) (E) Let $f(x + iy) = (x^2 - y) + i(x - y^2)$. For which complex numbers $z_0$ is $f$
   
   (a) Continuous at $z_0$?
   
   (b) Differentiable at $z_0$?
   
   (c) Analytic at $z_0$?

8. (bonus) §1.5 – Problem 28. (Hint: it’s easiest to think of $e^{z^2}$ as the composition of two mappings, and to look at the images step-by-step.)

9. (bonus) §2.1 – Problem 14. I want a careful proof by induction on $n$; no “…”’s! We shall use this formula later in the course.

10. (bonus) Suppose $f(z)$ is analytic in the complex plane and $f(x + iy) = u(x, y) + iv(x, y)$ where $v(x, y) = (u(x, y))^2$. Prove that $u$ and $v$ are both constants, hence $f = c$. (Hint: use Cauchy-Riemann at an arbitrary point $(x_0, y_0)$. At one stage of the proof you should consider separately the cases $u(x_0, y_0) = 0$ and $u(x_0, y_0) \neq 0$.)