1. (graded) §3.3 – 4a, 4c.
2. (graded) §3.3 – 11, 12.
3. (graded) (E) Find the first three terms of the Taylor series of \( f(z) = z^i \) at \( z = 1 + i \).

Unevaluated expressions such as \((1 + i)^i\) should not appear in your answer. (Here, \( z^i = e^{i \log(z)} \), where \( \log(z) \) is as before.) This could have been on an earlier homework; I wanted to complete your collection of old exam problems.

4. (graded) (E) Determine the linear fractional transformation \( T(z) = \frac{az + b}{cz + d} \) so that \( T(0) = 1 \), \( T(1) = i \) and \( T(\infty) = -1 \), and determine \( T(i) \).

5.&6. (graded) (E) Suppose

\[
T(z) = \frac{z + i}{z + 1}.
\]

a. Determine \( T(0), T(1), T(-1), T(i), T(-i), T(1 + i) \) and \( T(\infty) \).

b. Determine the images of the real axis, the imaginary axis and the unit circle under \( T \). (These might not be as simple as the examples given in class.)

c. Find (with explanation) the image of the first quadrant \( \{x + iy : x, y \geq 0\} \) under \( T \).

7. (graded) What is the name of the author of the textbook. This is not a trick question, but a way of making the homework shorter.

8. (bonus) Suppose \( f \) is an entire function satisfying \( 1 \leq |f(z)| \leq 2 \) for \( |z| = 1 \). Suppose there exists exactly one \( z_0 \in \mathbb{C} \) so that \( f(z_0) = (2 + i)z_0^2 \). Determine (with proof) \( f'(z_0) \).

(This is a twisted version of something you already know how to do.)

9. (bonus) Suppose \( f \) is analytic in \( |z| < 3 \), and \( f(-1) = f(1) = 0, f(0) = 4 \). Prove that there exists \( z_0 \) with \( |z_0| = 2 \) and \( |f(z_0)| \geq 12 \). Suppose further that \( f(2) = 12i \). Determine \( f(2i) \).

10. (bonus) Suppose

\[
T(z) = \frac{az + b}{cz + d}
\]

is a linear fractional transformation, and \( r_1, r_2, r_3, r_4 \) are distinct complex numbers with the property that \( T(r_1) = r_2, T(r_2) = r_1 \) and \( T(r_3) = r_4 \). What are the possible value or values for \( T(r_4) \)? (There are several different valid approaches to this problem.)