

428 HW1 Solutions Supplement, 10/2/17

#1

```
In[1] s[0] = 0; s[1] = 1; s[n_?EvenQ] := s[n] = s[n/2];
      s[n_?OddQ] := s[n] = s[(n+1)/2] + s[(n-1)/2]
```

```
In[2] s[10 022 017]
```

```
Out[2] 7379
```

```
In[3] s[2 102 017]
```

```
Out[3] 573
```

```
In[4] BaseForm[2 102 017, 2]
```

```
Out[4] BaseForm 1000000001001100000001_2
```

```
In[5] FromContinuedFraction[{1, 7, 2, 2, 1, 8, 1}]
```

```
Out[5]  $\frac{573}{505}$ 
```

```
In[6] s[2 102 018]
```

```
Out[6] 505
```

#2

```
In[7] ContinuedFraction[121/100]
```

```
Out[7] {1, 4, 1, 3, 5}
```

```
In[8] FromDigits[{1, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1}, 2]
```

```
Out[8] 15905
```

```
In[9] {s[15905], s[15906]}
```

```
Out[9] {121, 100}
```

#3

```
In[10] TableForm[Table[{r, s[2^r+1], s[2^r+2], s[2^r+3], s[2^r+4], s[2^r+5],
                       s[2^r+6], s[2^r+7], s[2^r+8], s[2^r+9], s[2^r+10]}, {r, 0, 10}]]
```

Out[10]: TableForm -

	1	2	3	4	5	6	7	8	9	10
0	1	2	1	3	2	3	1	4	3	5
1	2	1	3	2	3	1	4	3	5	2
2	3	2	3	1	4	3	5	2	5	3
3	4	3	5	2	5	3	4	1	5	4
4	5	4	7	3	8	5	7	2	7	5
5	6	5	9	4	11	7	10	3	11	8
6	7	6	11	5	14	9	13	4	15	11
7	8	7	13	6	17	11	16	5	19	14
8	9	8	15	7	20	13	19	6	23	17
9	10	9	17	8	23	15	22	7	27	20
10	11	10	19	9	26	17	25	8	31	23

#4

```
In[11] b[0] = 0; b[1] = 1; b[n_?EvenQ] := b[n] = -b[n/2];
      b[n_?OddQ] := b[n] = b[(n+1)/2] + b[(n-1)/2]
```

```
In[12] Table[b[n], {n, 0, 15}]
```

```
Out[12] {0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0, 1, -1, 0}
```

In[18] **Expand[x Product[(1 - x^(2^k) + x^(2^(k+1))), {k, 0, 5}]]**

Out[18] $x - x^2 + x^4 - x^5 + x^7 - x^8 + x^{10} - x^{11} + x^{13} - x^{14} + x^{16} - x^{17} + x^{19} - x^{20} + x^{22} - x^{23} + x^{25} - x^{26} + x^{28} - x^{29} + x^{31} - x^{32} + x^{34} - x^{35} + x^{37} - x^{38} + x^{40} - x^{41} + x^{43} - x^{44} + x^{46} - x^{47} + x^{49} - x^{50} + x^{52} - x^{53} + x^{55} - x^{56} + x^{58} - x^{59} + x^{61} - x^{62} + x^{64} - x^{66} + x^{67} - x^{69} + x^{70} - x^{72} + x^{73} - x^{75} + x^{76} - x^{78} + x^{79} - x^{81} + x^{82} - x^{84} + x^{85} - x^{87} + x^{88} - x^{90} + x^{91} - x^{93} + x^{94} - x^{96} + x^{97} - x^{99} + x^{100} - x^{102} + x^{103} - x^{105} + x^{106} - x^{108} + x^{109} - x^{111} + x^{112} - x^{114} + x^{115} - x^{117} + x^{118} - x^{120} + x^{121} - x^{123} + x^{124} - x^{126} + x^{127}$

In[21] **Expand[%18 - Sum[x^(3 k + 1) - x^(3 k + 2), {k, 0, 41}]]**

Out[21] $x^{65} - x^{66} + x^{68} - x^{69} + x^{71} - x^{72} + x^{74} - x^{75} + x^{77} - x^{78} + x^{80} - x^{81} + x^{83} - x^{84} + x^{86} - x^{87} + x^{89} - x^{90} + x^{92} - x^{93} + x^{95} - x^{96} + x^{98} - x^{99} + x^{101} - x^{102} + x^{104} - x^{105} + x^{107} - x^{108} + x^{110} - x^{111} + x^{113} - x^{114} + x^{116} - x^{117} + x^{119} - x^{120} + x^{122} - x^{123} + x^{125} - x^{126} + x^{127}$

HG

In[22] **TableForm[Table[Fibonacci[n, λ], {n, 0, 10}]]**

Out[22]: TableForm=

```

0
1
λ
1 + λ2
2 λ + λ3
1 + 3 λ2 + λ4
3 λ + 4 λ3 + λ5
1 + 6 λ2 + 5 λ4 + λ6
4 λ + 10 λ3 + 6 λ5 + λ7
1 + 10 λ2 + 15 λ4 + 7 λ6 + λ8
5 λ + 20 λ3 + 21 λ5 + 8 λ7 + λ9

```

In[23] **TableForm[Table[CoefficientList[Fibonacci[n, λ], λ], {n, 0, 10}]]**

Out[23]: TableForm=

```

1
0 1
1 0 1
0 2 0 1
1 0 3 0 1
0 3 0 4 0 1
1 0 6 0 5 0 1
0 4 0 10 0 6 0 1
1 0 10 0 15 0 7 0 1
0 5 0 20 0 21 0 8 0 1

```

In[26] **Sum[Binomial[9 - k, k] λ^(9 - 2 k), {k, 0, 5}]**

Out[26] $5 λ + 20 λ^3 + 21 λ^5 + 8 λ^7 + λ^9$

In[27] **RSolve[{f[n] == λ f[n - 1] + f[n - 2], f[0] == 0, f[1] == 1}, f[n], n]**

Out[27] $\left\{ \left\{ f[n] \rightarrow -\frac{2^{-n} \left(\left(\lambda - \sqrt{4 + \lambda^2} \right)^n - \left(\lambda + \sqrt{4 + \lambda^2} \right)^n \right)}{\sqrt{4 + \lambda^2}} \right\} \right\}$

In[28] **f[n] /. %27[[1]]**

$$\text{Out[28]} = \frac{2^{-n} \left((\lambda - \sqrt{4 + \lambda^2})^n - (\lambda + \sqrt{4 + \lambda^2})^n \right)}{\sqrt{4 + \lambda^2}}$$

In[29] **g[k_] := %28 /. n -> k**

In[30] **g[k]^2 - g[k-1] g[k+1]**

$$\text{Out[30]} = \frac{2^{-2k} \left((\lambda - \sqrt{4 + \lambda^2})^k - (\lambda + \sqrt{4 + \lambda^2})^k \right)^2}{4 + \lambda^2} - \frac{2^{-2k} \left((\lambda - \sqrt{4 + \lambda^2})^{-1 \cdot k} - (\lambda + \sqrt{4 + \lambda^2})^{-1 \cdot k} \right) \left((\lambda - \sqrt{4 + \lambda^2})^{1 \cdot k} - (\lambda + \sqrt{4 + \lambda^2})^{1 \cdot k} \right)}{4 + \lambda^2}$$

In[31] **Simplify[%]**

$$\text{Out[31]} = -4^{-k} \left(\lambda - \sqrt{4 + \lambda^2} \right)^k \left(\lambda + \sqrt{4 + \lambda^2} \right)^k$$

In[32] **FullSimplify[%]**

$$\text{Out[32]} = -4^{-k} \left(\lambda - \sqrt{4 + \lambda^2} \right)^k \left(\lambda + \sqrt{4 + \lambda^2} \right)^k$$

In[33] **Table[Simplify[%30 /. k -> j], {j, 0, 10}]**

Out[33] **{-1, 1, -1, 1, -1, 1, -1, 1, -1, 1, -1}**

In[34] **10! / (3600 * 24)**

Out[34] **42**

10! seconds = 6 weeks exactly!