Math 428, Fourth worksheet, 9/22/17

1. We have used the general fact

$$\sum_{n=2^{r+1}}^{2^{r+2}-1} f(n) = \sum_{n=2^r}^{2^{r+1}-1} f(2n) + \sum_{n=2^r}^{2^{r+1}-1} f(2n+1)$$

along with the definitions

$$A(r) = \sum_{n=2^r}^{2^{r+1}-1} s(n)^2, \quad B(r) = \sum_{n=2^r}^{2^{r+1}-1} s(n)s(n+1)$$

to derive the homogenous system of recurrences:

$$A(r+1) = 3A(r) + 2B(r); \qquad B(r+1) = 2A(r) + 2B(r).$$

Now define

$$C(r) = \sum_{n=2^r}^{2^{r+1}-1} s(n)s(n+2)$$

a. Recall that $s(2^r+1) = r+1$ and be careful with the indices to express

$$\sum_{n=2^r+1}^{2^{r+1}} s(n)s(n+1)$$

in terms of $B(r)$.

b. Using the earlier method, find an expression for $C(r+1)$ in terms of $A(r), B(r), C(r)$. Big hint: it will not be homogeneous. You can check your work against the attached table on the previous page.