Math 428 Worksheet 9/8/17

1. We saw earlier that if (F_n) is the Fibonacci sequence, then

$$\sum_{n=0}^{\infty} F_n x^n = \frac{x}{1-x-x^2} = F(x).$$

$$F_0=0,\ F_1=1,\quad F_{n+2}=F_{n+1}+F_n,\quad n\geq 0$$

(a) Determine the generating function for (F_{n+1}), ie $\sum_{n=0}^{\infty} F_{n+1}x^n$

(Hint: it's easy!)

(b). The Lucas sequence is defined by

$$L_0=2,\quad L_1=1,\quad L_{n+2}=L_{n+1}+L_n,\quad n\geq 0.$$

Determine the generating fn

$$L(x)=\sum_{n=0}^{\infty} L_n x^n.$$

(Hint: multiply by some factor $1-x-x^2$.)

(c) Use (a) and (b) to find α, β so that

$$L_n = \alpha \cdot F_n + \beta F_{n+1}$$

2. Extend the Fibonacci sequence to F_n for $n \in \mathbb{Z}$ by assuming the recurrence.

Thus $F_1 = F_0 + F_{-1} \Leftrightarrow 1 = 0 + F_{-1} \Rightarrow F_{-1} = 1$

$F_0 = F_{-1} + F_{-2} \Leftrightarrow 0 = F_{-1} + F_{-2} \Rightarrow F_{-2} = -1$

Guess a formula for F_{-n}.