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In[66]= **TableForm**[**Table**[{r, C1[r+1] - C1[r] - A[r] - 3 B[r] - 1}, {r, 0, 10}]]

Out[66]/TableForm=

0	0
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0

In[68]= **TableForm**[**Table**[{r, A[r], B[r], C1[r], C1[r+2] - 5 C1[r+1] + 2 C1[r] + 2 r + 6}, {r, 0, 10}]]

Out[68]/TableForm=

0	1	1	2	0
1	5	4	7	0
2	23	18	25	0
3	105	82	103	0
4	479	374	455	0
5	2185	1706	2057	0
6	9967	7782	9361	0
7	45465	35498	42675	0
8	207391	161926	194635	0
9	946025	738634	887805	0
10	4315343	3369318	4049733	0

In[69]= **FullSimplify**[**RSolve**[{a[n] = 5 a[n-1] - 2 a[n-2], a[0] = 1, a[1] = 5}, a[n], n]]

Out[69]=  $\left\{ \left\{ a[n] \rightarrow \frac{1}{34} \left( (17 - 5\sqrt{17}) \left( \frac{1}{2} (5 - \sqrt{17}) \right)^n + \left( \frac{1}{2} (5 + \sqrt{17}) \right)^n (17 + 5\sqrt{17}) \right) \right\} \right\}$

In[70]= **FullSimplify**[**RSolve**[{a[n] = 5 a[n-1] - 2 a[n-2], a[0] = 1, a[1] = 4}, a[n], n]]

Out[70]=  $\left\{ \left\{ a[n] \rightarrow \frac{1}{34} \left( (17 - 3\sqrt{17}) \left( \frac{1}{2} (5 - \sqrt{17}) \right)^n + \left( \frac{1}{2} (5 + \sqrt{17}) \right)^n (17 + 3\sqrt{17}) \right) \right\} \right\}$

In[71]= **FullSimplify**[**RSolve**[{a[n] = 5 a[n-1] - 2 a[n-2] - 2 n - 6, a[0] == 2, a[1] = 7}, a[n], n]]

Out[71]=  $\left\{ \left\{ a[n] \rightarrow \frac{1}{17 (5 + \sqrt{17})} 2^{-1-2n} \left( (2 (5 + \sqrt{17}))^n (85 + 37\sqrt{17}) + 2^n (-4 (5 - \sqrt{17})^n (85 + 22\sqrt{17}) + 17 \times 2^n (5 + \sqrt{17}) (7 + 2n)) \right) \right\} \right\}$

In[72]= **u[n\_] := c1 A[n] + c2 B[n] + c3 n + c4**

In[73]= **Table**[u[n], {n, 0, 3}]

Out[73]= {c1 + c2 + c4, 5 c1 + 4 c2 + c3 + c4, 23 c1 + 18 c2 + 2 c3 + c4, 105 c1 + 82 c2 + 3 c3 + c4}

In[74]= **Table**[C1[n], {n, 0, 3}]

Out[74]= {2, 7, 25, 103}

In[75]:= **Solve[%73 = %74]**

Out[75]=  $\left\{ \left\{ c1 \rightarrow \frac{5}{2}, c2 \rightarrow -2, c3 \rightarrow 1, c4 \rightarrow \frac{3}{2} \right\} \right\}$

In[77]:= **TableForm[Table[{r, A[r], B[r], C1[r], (5/2) A[r] - 2 B[r] + r + (3/2)}, {r, 0, 10}]]**

Out[77]/TableForm=

0	1	1	2	2
1	5	4	7	7
2	23	18	25	25
3	105	82	103	103
4	479	374	455	455
5	2185	1706	2057	2057
6	9967	7782	9361	9361
7	45465	35498	42675	42675
8	207391	161926	194635	194635
9	946025	738634	887805	887805
10	4315343	3369318	4049733	4049733

In[80]:= **TableForm[Table[{r, A[r], B[r], N[B[r] / A[r]}], {r, 0, 10}]]**

Out[80]/TableForm=

0	1	1	1.
1	5	4	0.8
2	23	18	0.782609
3	105	82	0.780952
4	479	374	0.780793
5	2185	1706	0.780778
6	9967	7782	0.780777
7	45465	35498	0.780776
8	207391	161926	0.780776
9	946025	738634	0.780776
10	4315343	3369318	0.780776

In[87]:= **TableForm[Table[{r, A[r], C1[r], N[C1[r] / A[r]}], {r, 0, 12}]]**

Out[87]/TableForm=

0	1	2	2.
1	5	7	1.4
2	23	25	1.08696
3	105	103	0.980952
4	479	455	0.949896
5	2185	2057	0.941419
6	9967	9361	0.939199
7	45465	42675	0.938634
8	207391	194635	0.938493
9	946025	887805	0.938458
10	4315343	4049733	0.93845
11	19684665	18473031	0.938448
12	89792639	84265663	0.938447

In[81]:= **N[(17 + 3 Sqrt[17]) / (17 + 5 Sqrt[17])]**

Out[81]= 0.780776

In[82]= **FullSimplify**[(17 + 3 Sqrt[17]) / (17 + 5 Sqrt[17])]

$$\text{Out[82]} = \frac{1}{4} (-1 + \sqrt{17})$$

In[84]= **FullSimplify**[5/2 - 2 \* %82]

$$\text{Out[84]} = 3 - \frac{\sqrt{17}}{2}$$

In[85]= **N**[%]

Out[85]= 0.938447

In[86]= **C1**[18] / **A**[18]

$$\text{Out[86]} = \frac{759\,148\,637\,741}{808\,941\,246\,863}$$

In[89]= **ContinuedFraction**[%88]

Out[89]= {0, 1, 15, 4, 16, 4, 16, 2, 1, 2, 1, 1, 3, 2, 1, 17, 1, 4, 1, 2, 80, 2}

In[90]= **FromContinuedFraction**[{4, 16, x}]

$$\text{Out[90]} = \frac{4 + 65 x}{1 + 16 x}$$

In[91]= **Solve**[x = %90]

$$\text{Out[91]} = \left\{ \left\{ x \rightarrow \frac{1}{2} (4 - \sqrt{17}) \right\}, \left\{ x \rightarrow \frac{1}{2} (4 + \sqrt{17}) \right\} \right\}$$

In[93]= **Simplify**[**FromContinuedFraction**[{0, 1, 15, (4 + Sqrt[17]) / 2}]]

$$\text{Out[93]} = 3 - \frac{\sqrt{17}}{2}$$

