Math 428, Second worksheet, 9/11/17

1. Recall that

$$s(0) = 0, s(1) = 1; \qquad s(2n) = s(n), \quad s(2n+1) = s(n) + s(n+1) \text{ for } n \geq 1.$$

Let

$$t(n) = \sum_{k=0}^{n} s(k)$$

See the table.

1a. Find a formula which relates $t(2n)$ to $t(n)$ and $s(n)$. Hint:

$$\sum_{k=0}^{2n} s(k) = \sum_{k=0}^{n} s(2k) + \sum_{k=0}^{n-1} s(2k+1)$$

1b. Recall that

$$\sum_{k=a}^{b}{}^{*} f(k) = \sum_{k=a}^{b} f(k) - \frac{1}{2}\left(f(a) + f(b)\right).$$

Using your result from 1a, find a simpler formula for

$$t^*(2n) = \sum_{k=0}^{n}{}^{*} s(k)$$

in terms of $t^*(n)$.

2. We already know that $s(n)$ is periodic mod 2, with $s(n) \equiv 0, 1, 1 \pmod 2$ as $n \equiv 0, 1, 2 \pmod 3$.

2a. Determine the behavior of $t(n) \pmod 2$, depending on $n \pmod 3$.

2b. Over the weekend, I ran a computer experiment which showe that there are no $n < 10^5$ for which $t(n) \equiv 3 \pmod 4$. Then I proved it with a short proof. Find my proof! (I'm leaving this without hints, but am prepared to give them in class as needed.)