1. We have \( f : [1, 1] \to \mathbb{R} \) if \( x < 17 \) for all \( x \), and 
\[
g(x) = x f(x), \quad \text{so} \quad g(x) < 17 x.
\]
Since \( g(0) = 0 \) and \( f(0) = 0 \), so 
Show that \( g \) is continuous, do we use the \( \varepsilon - \delta \) definition.

Suppose \( \varepsilon > 0 \) is given 
Let \( \delta = \frac{\varepsilon}{17} \). Then 
\[
x < 0 (\varepsilon) \Rightarrow x < 17 \delta = \frac{\varepsilon}{17}
\]
so 
\[
|g(x) - g(0)| = |g(x)| < 17 \delta = \frac{\varepsilon}{17}.
\]
The real theorem of course is that \( f(x) \leq M \) in an open interval.

The corollary of this is that \( f(x) \leq M \) in an open interval.

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4. Observe \( N = 2 \cdot \frac{\varepsilon}{5} + 3 \cdot \frac{\varepsilon}{5} = 2 \cdot \frac{\varepsilon}{5} + 3 \cdot \frac{\varepsilon}{5} < \varepsilon \)

My choice of \( \frac{\varepsilon}{5} \) will be determined by \( \varepsilon \).

Let \( \varepsilon_n = \frac{\varepsilon}{5^n} \).

For any \( \varepsilon > 0 \), take \( n \) such that \( \frac{\varepsilon}{5^n} < \varepsilon \).

We already know that
\[
2f_n + 3g_n = 2f + 3g \text{ on } [1, 7]
\]

Is it uniform?

Consider \( \sum_{n=1}^{N} f_n(x) \).

\[
\left| f_n(x) - f(x) \right| < \frac{\varepsilon}{5^n}, \quad x \in [1, 7], \quad n \leq N
\]

\[
\left| g_n(x) - g(x) \right| < \frac{\varepsilon}{5^n}, \quad x \in [1, 7], \quad n \leq N
\]

So if \( n \geq N \geq N(x, N) \),
\[
\left| f_n(x) - f(x) \right| + \left| g_n(x) - g(x) \right| < 2 \cdot \frac{\varepsilon}{5^n} + 3 \cdot \frac{\varepsilon}{5^n} = \varepsilon
\]

So convergence is uniform.

5a. So for each \( p \in E \) we have an open set
\[
B(p, \varepsilon) : x \in B(p, \varepsilon) \Rightarrow \left| f(p) \right| < \varepsilon
\]

(M depends on \( p \)).

And \( E = \bigcup_{p \in E} B(p, \varepsilon) \)

Since \( E \) is compact, it is contained in a finite union
\[
E = \bigcup_{j=1}^{N} B(p_j, \varepsilon)
\]

cd. If \( \left| f(p) \right| < M \), then
\[
\left| f(x) \right| < \max(M, \ldots, M_n)
\]

It follows that for all \( x \in E \),
\[
\left| f(x) \right| < \max(M_1, \ldots, M_n).
\]

This is an example of the usefulness of compact spaces.

5b. Let \( C = \mathbb{R} \), \( f(x) = x \).

For any \( x \in C \), take \( r = 1 \).

\[
|f(x)| \leq |x| + 1 \quad \forall x \in \mathbb{R}
\]

So with \( M = |x| + 1 \), \( f \) is locally bounded. However, \( f \) is not bounded on \( \mathbb{R} \) itself.

6. (Exercises)

My sequence is \( \{x_n \} \):
\[
x_1, x_2, x_3, \ldots, a_1, a_2, a_3, \ldots
\]

(\( a_n \) because \( a_1, a_2, \ldots \in \mathbb{R} \)).

Appears in \( (b) \).

(b) Assume \( l \) is a cluster point.

\( c) \) If \( x < a_1 \) and \( \delta = \frac{a_1 - x}{2} \), then no points of the sequence satisfy \( x - \delta < x \).

(d) If \( a_n \to L \) and \( y > L \); otherwise,
\[
y \neq L, \quad \delta = \frac{y - L}{2}, \quad \text{no points of the sequence are within } \delta \text{ of } y.
\]

(e) Suppose \( z \) is a cluster point.
\[
\text{Consider } (z - \delta, z + \delta)
\]

Thus is an example of the usefulness of compact spaces.
The points of \((0,1)\) make

**Theorem 1.** There is no \(A\) not coinciding with \(A\).

**Proof.** Let \(A\) be the union of \(A_1, A_2, \ldots, A_n\), where each \(A_i\) is a subset of \(A\). Then \(A\) is the union of \(A_1, A_2, \ldots, A_n\).

- \(A_1\), \(A_2\), \(\ldots\), \(A_n\) are all subsets of \(A\).
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**Corollary.** There is no \(A\) not coinciding with \(A\).

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**Theorem 2.** If \(A\) is not coinciding with \(A\), then there is no \(A\) not coinciding with \(A\).

**Proof.** Let \(A\) be the union of \(A_1, A_2, \ldots, A_n\), where each \(A_i\) is a subset of \(A\). Then \(A\) is the union of \(A_1, A_2, \ldots, A_n\).

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