1a. For any \( x \in (-1, 1) \), this series is absolutely convergent, by Leibniz's theorem.

Thus, as in the book, we may separate the sum into two pieces.

\[
\sum_{n=0}^{\infty} (-1)^n x^{2n} + \sum_{n=0}^{\infty} (-1)^n x^{2n+1} = \sum_{n=0}^{\infty} (-1)^n (x^2)^n + \sum_{n=0}^{\infty} (-1)^n x^{2n+1} = \frac{1}{1-x^2} + \frac{x}{1-x^2} = \frac{1}{1-x^2}.
\]

1b. Since the convergence is uniform on \( [-1,1] \),

\[
\int_1^x (1+x^2+t^2-\ldots) \, dt = \int_1^x (1+t^2) \, dt.
\]

\[
x + \frac{x^3}{2} + \frac{x^5}{3} + \ldots = \arctan x + \frac{1}{2} \log(1+x^2)
\]

It is plausible then to say that

\[
1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \ldots = \arctan 1 + \frac{1}{2} \log 2
\]

\[
= \frac{\pi}{4} + \frac{\log 2}{2}
\]

To make this rigorous (which was asked):

Argue as on p.155 of no book.

2. My example (Choo, who lets possibilities). Any \( a \),

\[
a_n = \frac{a^n}{n!}, \quad a_{n+1} = \frac{a^{n+1}}{(n+1)!}, \ldots, \quad a_{n+k} = \frac{a^{n+k}}{(n+k)!},
\]

Thus, \( a = \frac{1}{a} \) and \( \frac{\log n}{n} \) do not converge.

2. My example (Choo, who lets possibilities).

\[
a_0 = \frac{1}{a} \quad a_1 = \frac{a}{a} \quad a_2 = \frac{a_1}{a} \quad a_3 = \frac{a_2}{a} \quad a_4 = \frac{a_3}{a} \quad a_5 = \frac{a_4}{a} \quad a_6 = \frac{a_5}{a} \quad a_7 = \frac{a_6}{a} \quad a_8 = \frac{a_7}{a} \quad a_9 = \frac{a_8}{a} \quad a_{10} = a_9 = \frac{a_9}{a} = \frac{a_8}{a} = \frac{a_7}{a} = \frac{a_6}{a} = \frac{a_5}{a} = \frac{a_4}{a} = \frac{a_3}{a} = \frac{a_2}{a} = \frac{a_1}{a} = \frac{a_0}{a} = \frac{1}{a} = a.
\]

3. \( \int_1^x e^{at} \, dt = \frac{e^{at} - a}{a^2} \).

\( f(t) = e^{at} \) is continuous at

\( x \in \mathbb{R} \).

To make this rigorous (which was asked):

\[
\int_1^x \frac{\partial (e^{at} \, dt)}{\partial t} \, dt = \frac{\partial}{\partial t} \left( \frac{e^{at} - a}{a^2} \right)
\]

\[
= \frac{a(e^{at} - a)}{a^2} = \frac{a(e^{at} - a)}{a^2}.
\]

2. My example (Choo, who lets possibilities).

\[
\int_1^x e^{at} \, dt = \left. \frac{e^{at} - a}{a^2} \right|_1^x
\]

\[
= \frac{e^{ax} - a}{a^2} - \frac{e^a - a}{a^2}.
\]

\( e^{-a} = \frac{1}{e^a} \) and \( a^2 \) do not converge.

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\]

\( e^{-a} = \frac{1}{e^a} \) and \( a^2 \) do not converge.
4a. \[ \sum_{n=0}^{\infty} \left( -\frac{3}{2} \right)^n \frac{2}{2n+1} \]

Nice alternating series, but

\[ \frac{n+2}{2n+1} \rightarrow \frac{1}{2} \quad \text{so series diverges} \]

4b. \[ \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} \]

Some tests:

If you prefer the root test:

\[ \left( \frac{1}{n\sqrt{n}} \right)^n = \frac{1}{n^n} \rightarrow 0 \]

10. \[ \sum_{n=1}^{\infty} \frac{1}{n^n} \]

10. \[ \frac{10^n}{2n^2} \rightarrow \frac{10}{2n^2} \rightarrow 0 \quad \text{as well} \]

4c. By ratio test: \[ n \xrightarrow{n \to \infty} \]

so \[ \sum_{n=0}^{\infty} \frac{1}{n!} \]

either both converge or both diverge, so sure \[ \frac{1}{\sqrt{n}} \]

Diverges, so does

[mathematical expressions]

Ungladd Lebifassinsmase

In a grade