Note: This material will be covered on the third exam.


3. (Extra credit 1.5 pts) Evaluate
\[ \lim_{n \to \infty} \left( \prod_{j=1}^{n} \left( \frac{n + j}{n} \right)^{\frac{n+j}{n^2}} \right) \]
Essential hint: take the logarithm of what you see. Think Riemann.

4. (E) Let \( F(x) \) be a continuous, strictly increasing function defined on \([0, 1]\) satisfying
\[ \frac{x}{2} \leq F(x) \leq 2x, \]
and let \( f_n(x) = F(x^n) \).

a. For \( x \in [0, 1] \), compute \( \lim_{n \to \infty} f_n(x) \). It will be helpful to distinguish \( x \in [0, 1) \) and \( x = 1 \), but \( \epsilon/\delta \) arguments are unnecessary.

b. Use the comparison test to show that \( \sum_{n=1}^{\infty} f_n(x) \) converges for \( x \in [0, 1) \).

c. Show that the convergence is uniform on \([0, \frac{1}{2}]\) but not on \([0, 1)\).

5. (E) Determine (with explanation) whether each of the following infinite series converge.
\[ \sum_{n=0}^{\infty} \frac{10^n \cdot n!}{(2n)!}, \quad \sum_{n=0}^{\infty} \frac{(424)^n}{n!}, \quad \sum_{n=0}^{\infty} (-1)^n \frac{(10)^n}{2^n}, \quad \sum_{n=1}^{\infty} \frac{\cos(e^n)}{n^{1.01}}. \]

(As a hint on the first one, observe that \((2(n+1))!\) is not the same as \((2n + 1)!\).)

6. (E) Let
\[ f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n \cdot 4^n}. \]

a. Determine the interval of convergence for this power series, including a discussion of the endpoints.

b. Express \( f' \) as a simple rational function.

c. Using b. and your knowledge of simple integration, or any correct method, compute
\[ \sum_{n=1}^{\infty} \frac{1}{n \cdot 4^n}. \]