Math 417 **Homework 1** **Due Friday, January 25, 2019**

It's ok to work together, but don't copy without comprehension. The grader and I can often detect this! The symbol ($\mathcal{E}$) means that at least part of this problem is a former exam question. No ungraded problems this week.

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1. ($\mathcal{E}$) Compute $g = \gcd(40, 65)$ by any correct method, and find two integers m and n so that $g = 40m + 65n$.

2a. Determine $(\mathbb{Z}/14\mathbb{Z})^*$. (Hints: $14 = 2 \cdot 7$ and this set has 6 elements. It is ok in this problem to write your solution with, say "1" standing in for $[1]_{14}$.)
2b. Write down the multiplication table for $(\mathbb{Z}/14\mathbb{Z})^*$.

2c. Which (if any) elements a have the property that the order of a is 6? That is, $a^6 \equiv 1 \mod 14$, but $a^j \not\equiv 1 \mod 14$ for $j \in \{1, 2, 3, 4, 5\}$.

3. Find by any correct and explained method all integers x with the property that

$$x \equiv 2 \mod 5,$$
$$x \equiv 6 \mod 7.$$

4. ($\mathcal{E}$) Given the arithmetic facts that $11 * 37 = 407$, $11 \mid 10^2 - 1$ and $37 \mid 10^3 - 1$ explain why you know that

$$\frac{1}{1407}$$

will have a decimal expansion that repeats after at most 6 digits, even if you don't calculate it. (This actually is a proof question; you don't have to confirm the arithmetic.)

5 ($\mathcal{E}$). The following is the multiplication table of a group G. You may assume this is a group, **you are not asked to prove this!** All questions can be answered by reading the table.

	U	V	W	X
U	V	X	U	W
V	X	W	V	U
W	U	V	W	X
X	W	U	X	V

5a. Which element is the identity in the group?
5b. Determine the inverses of u, v, w, x.

6. ($\mathcal{E}$) Suppose $\gcd(a, b) = 4$ and $\gcd(a, c) = 6$. Let $\gcd(b, c) = g$.
6a. What can you say about the power of 2 that divides g?
6b. What can you say about the power of 3 that divides g?
6c. What can you say about the power of 5 that divides g?